MH 408: Calculus of Higher Dimension: Test 3

Please box your answer. Show work for full credit.

(1.) (15pts) Calculate the integral below:

$$\int_0^1 \int_1^x \int_{e^y}^{x+y} 2z \, dz \, dy, dx$$

(2.) (10pts) Calculate $\frac{\partial(x,y,z)}{\partial(\alpha,\beta,\gamma)}$ given that $x = e^{\alpha+\beta}$ and $y = e^{\beta-\alpha}$ and $z = \sin^2(\gamma^2)$. Simplify your answer.

(3.) (10pts) Calculate $\displaystyle\int_0^1 \int_y^1 e^{x^2} \, dx \, dy$.

(4.) (10pts) Find the volume bounded by $x + y/2 + z/3 = 1$ and $x \geq 0$, $y \geq 0$ and $z \geq 0$.

(5.) (10pts) Let $f(x,y) = xy$ and let D be the quarter disk of radius R in the quadrant where $x \geq 0$ and $y \geq 0$. Calculate $\iint_D f \, dA$.

(6.) (10pts) Calculate the integral below by changing to spherical coordinates:

$$\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} (x^2+y^2) \, dz \, dy \, dx$$

(7.) (10pts) Find the volume of the solid bounded by $x^2 + y^2 = 3z^2$ and $z = h$ where $h > 0$.

(8.) (15pts) Let B be the hemisphere formed by $x^2 + y^2 + z^2 \leq R^2$ and $z \geq 0$. If the mass density $\delta = \frac{dm}{dV} = k\rho^2$ and B has a total mass of M then find the moment of inertia for B about the z-axis (recall we said $I = \iiint_B (x^2 + y^2) dm$). Do not use k in your answer, instead present the result in terms of M and R.

(9.) (10pts) Find the power series centered at $(0,0,0)$ for $f(x,y,z) = \dfrac{x\cos(y)}{1-z}$ up to second order.

(10.) (15pts) Let $f(x,y) = x^3 + y^3 - 3xy$. Find any critical points for f and use the second derivative test for functions of two-variables to classify the nature of each critical point as min./max/ or saddle.

(11.) (15pts) Let $f(x,y) = x^3 - 3x - y^2$. Use calculus to find the absolute extreme values of f in the half-disk $x^2 + y^2 \leq 9$ where $x \geq 0$. Draw a picture to explain your analysis.