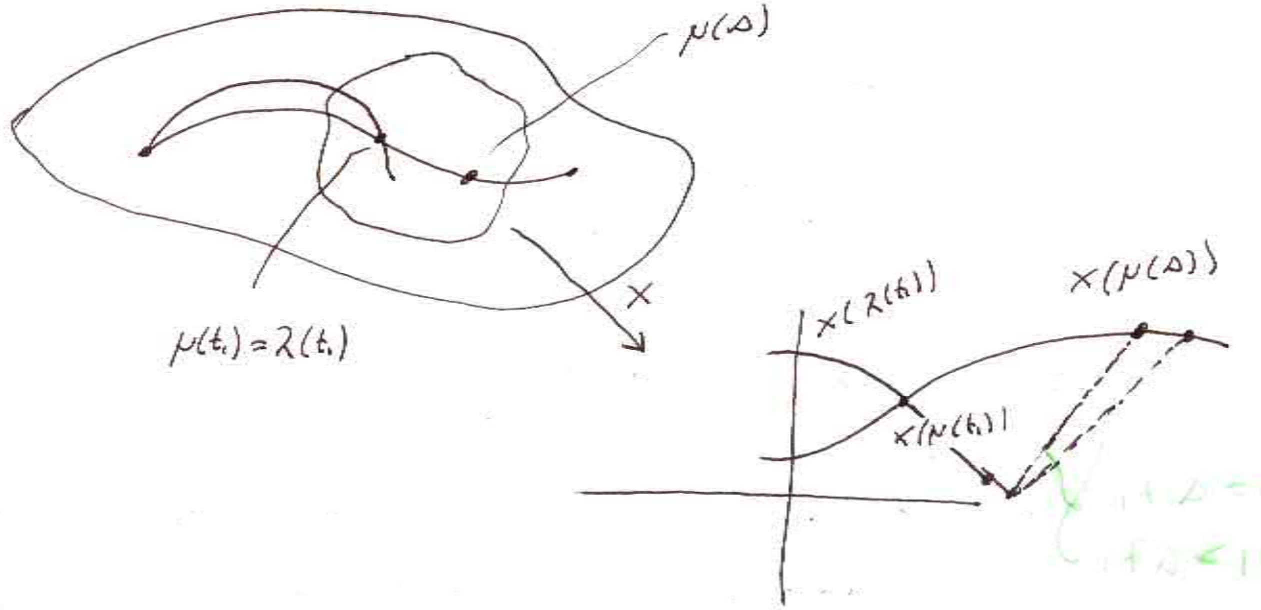
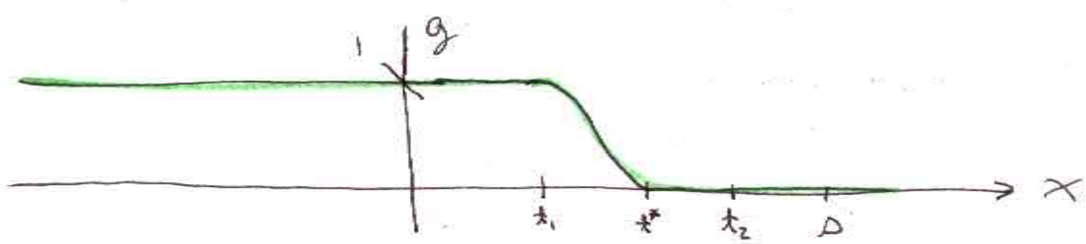
2/13/2001

Now consider the function $x \circ \gamma$



Choose $t^*, t_2 \ni t_1 < t^* < t_2 < t_1 + \delta < s$

Define $g : \mathbb{R} \longrightarrow \mathbb{R}$

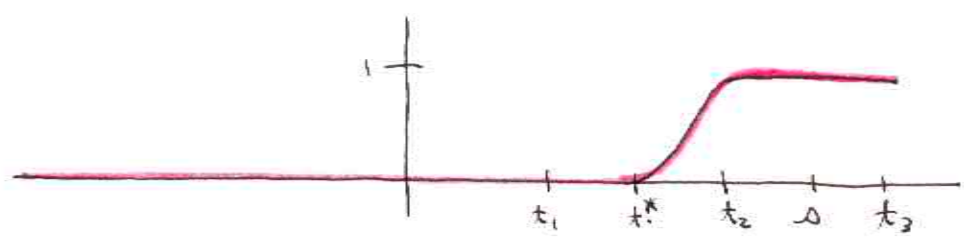


Then $g(t)x(\gamma(t))$ is smooth for $(t_1 - \delta, \infty)$ with value zero for $t \geq t^*$

If $s = 1$ then we define $t_3 = s$. If $s < 1$ let $s < t_3 < 1$. We show

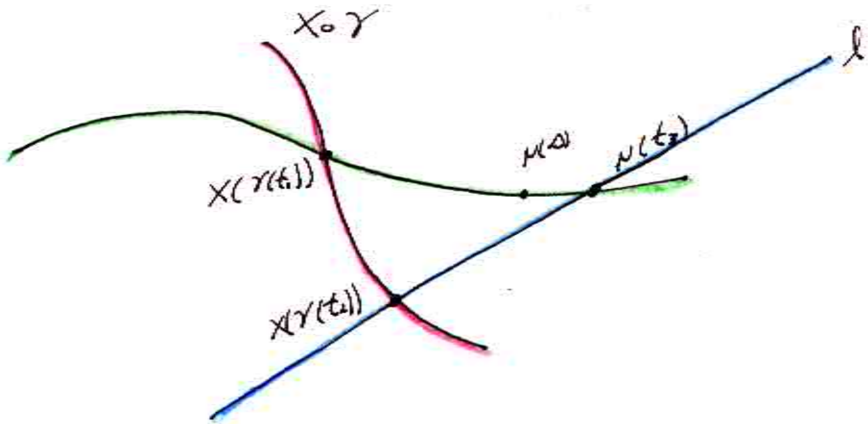
$\exists$ smooth $\tilde{\lambda} : [0, t^3] \longrightarrow M$ such that $\tilde{\lambda}(0) = \mu(0)$, $\tilde{\lambda}(t_3) = \mu(t_3)$

which will imply that $t_3 \in V$ and $t_3 > s = \text{lub}(V)$. We define $h : \mathbb{R} \to$



Define $l : \mathbb{R} \to \mathbb{R}^n$ to be a ~~staight~~ smooth function whose graph is a straight line in $\mathbb{R}^m$ such that $l(t_2) = x(\gamma(t_2))$ and $l(t_3) = x(\mu(t_3))$

2/13/2007



$\boxed{\mathbb{R}^m}$

$h(t)\,\ell(t)$ is defined on all of $\mathbb{R}$. Let $\psi(t) = g(t)\,x(\gamma(t)) + h(t)\,\ell(t)$ for all $t > t_1 - \delta$. Now note the properties of ψ

$$\text{Claim: } \Rightarrow \left.\begin{array}{l} \psi \,|\, (t_1-\delta, t_1] = (x\circ\gamma)(t_1-\delta, t_1] \\ \psi \,|\, [t_1, t^*] = g(t)\,x(\gamma(t)) \\ \psi \,|\, [t^*, t_2] = h(t)\,\ell(t) \end{array}\right\} \text{from defining graphs of functions}$$

$$\psi \,|\, [t_2, \infty) = \ell(t)$$

Now $x^{-1}\circ\psi$ is smooth in $(t_1-\delta, t_3]$ and agrees with γ on $(t_1-\delta, t_3]$. Finally we can define $\tilde{\gamma}$

$$\tilde{\gamma}(t) = \left\{\begin{array}{ll} \gamma(t) & 0 \le t \le t_1 \\ x^{-1}(\psi(t)) & t_1 \le t \le t_3 \end{array}\right\}$$

Now $\tilde{\gamma} : [0, t_3] \longrightarrow M$ smooth, $\tilde{\gamma}(0) = \gamma(0) = \mu(0)$

$$\tilde{\gamma}(t_3) = x^{-1}(\psi(t_3)) = x^{-1}\big(h(t_3)\,\ell(t_3)\big)x(\gamma(t_2)) = x^{-1}\big(x(\mu(t_3))\big) = \mu(t_3)$$

$$\Rightarrow t_3 \in V \text{ and } t_3 > \mathrm{lub}(V) \longrightarrow\longleftarrow$$

$\boxed{\text{QED}}$

$$\boxed{F^\infty(P) = C_p^\infty(M)}$$

If M is a manifold, we say that $f \in C_p^\infty(M)$ iff f is a smooth function from an open subset U of M into $\mathbb{R}$ such that $P \in U$

Now let $f, g \in C_p^\infty(M)$ then

$$(f+g)(x) = f(x) + g(x)$$
$$(cf)(x) = cf(x)$$
$$(fg)(x) = f(x)g(x)$$

If f is defined on U and g is defined on V then $\text{dom}(f+g) = U \cap V$ and $\text{dom}(cf) = \text{dom}(f)$ exc...

problem 1.6.2
Extra if
we do well

$$\hat{0}(x) = 0$$
$$f + \hat{0} = f = \hat{0} + f$$
$$f + (-1)f = \hat{0}|_U \neq \hat{0}$$ domains a problem

We say that X is a first order linear differential operator at $p \in M$ iff $X : C_p^\infty(M) \longrightarrow \mathbb{R}$ such that

1.) $X(f+g) = X(f) + X(g)$
2.) $X(cf) = c\,X(f)$
3.) $X(fg) = f(P)\,X(g) + g(P)\,X(f)$

(Another view of Tangent vectors)

Let (U, x) be a chart on M. We Define the following

$$\left.\frac{\partial}{\partial x^i}\right|_p \equiv \frac{\partial}{\partial x^i}(p) \qquad \text{(notation)}$$

$$\frac{\partial}{\partial x^i}(p)(f) \equiv \frac{\partial(f \circ x^{-1})}{\partial u^i}(x(p))$$

Which the $(u^1, u^2, \ldots, u^m) \in \mathbb{R}^m$ and $x(U) \underset{open}{\subseteq} \mathbb{R}^m$

Notice that $\frac{\partial f}{\partial x^i}(p)$ has no particular meaning on its own.

Properties

$$(f+g) \circ x^{-1} = f \circ x^{-1} + g \circ x^{-1}$$

$$(fg) \circ x^{-1} = (f \circ x^{-1})(g \circ x^{-1})$$

$$(cf) \circ x^{-1} = c(f \circ x^{-1})$$

$$\left.\frac{\partial}{\partial x^i}\right|_p (f+g) = \left.\frac{\partial}{\partial x^i}\right|_p (f) + \left.\frac{\partial}{\partial x^i}\right|_p (g)$$

Rod Length L

$\boxed{1.2.6}$ $(u_1, u_2, u_3, u_4, u_5, u_6) \in \mathbb{R}^6$

$L(a_1, a_2, a_3)$

Show manifold same as $\mathbb{R}^3 \times S^2$

So he wants a mapping $\varphi : \mathbb{R}^3 \times S^2 \longrightarrow \mathbb{R}^6$

Show φ a 1-1 mapping. We set up a 1-1 correspondence between configuration space and manifold

I. TEST - 2 weeks from 2/15/2001

II. Homework Next Thursdo

1.7.1
1.7.2
1.8.3
1.8.4

$\boxed{1.2.9}$

EASY : $\mathbb{R}^3 \times S^2 \times (L_1, L_2)$

Projective Plane Problems

$$\mathbb{R}P^2 = \{\{-P, P\} \mid P \in S^2\}$$

$$P \in S^2 \iff x(P)^2 + y(P)^2 + z(P)^2 = 1$$

$$\mathbb{R}P^2 = \left\{\{(x, y, z), (-x, -y, -z)\} \,\middle|\, x^2 + y^2 - z^2 = 1\right\}$$

$$\mu_1(\{+P, -P\}) = \left(\frac{y}{z}, \frac{z}{x}\right) = \mu_1\left(\{(x, y, z), (-x, -y, -z)\}\right)$$

$x^2 + y^2 + z^2 = 1$

$F : S^2 \longrightarrow \mathbb{R}P^2$ $\quad \chi$ a chart on S^2

$$\begin{tikzcd} S^2 \arrow[r, "F"] \arrow[d, "x"'] & \mathbb{R}P^2 \arrow[d, "\mu_1"] \\ {} \arrow[r, "\chi^{-1} \circ F \circ \mu_1"'] & {} \end{tikzcd}$$

(smooth ?)

$$C_p^\infty(M) = \tilde{\mathcal{F}}^\infty(P) \quad : \quad \text{Book's Notation}$$

$$X_p : C_p^\infty M \longrightarrow \mathbb{R}$$

$$X_p(c_1 f_1 + c_2 f_2) = c_1 X_p(f_1) + c_2 X_p(f_2)$$

$$X_p(fg) = f(P)\, X_p(g) + g(P)\, X_p(f)$$

Notation

$$\left.\frac{\partial}{\partial x^i}\right|_p = \frac{\partial}{\partial x^i(P)} = \partial_i^x$$

$$\frac{\partial}{\partial u^i} = \partial_i$$

Differentiation with respect to x^i from chart

$$\boxed{\text{Def}^{n}/ \quad \frac{\partial}{\partial x^i}(P)(f) \equiv \frac{\partial(f \circ x^{-1})}{\partial u^i}(x(P)) = \partial_i(f \circ x^{-1})(x(P))}$$

$$(fg) \circ x^{-1} = (f \circ x^{-1})(g \circ x^{-1})$$

$$\frac{\partial}{\partial x^i}(P)(fg) = \frac{\partial(fg \circ x^{-1})}{\partial u^i}(x(P)) = \frac{\partial}{\partial u^i}\left((f \circ x^{-1})(g \circ x^{-1})\right)(x(P))$$

$$= (f \circ x^{-1})(x(P)) \frac{\partial}{\partial u^i}(g \circ x^{-1})(x(P)) + (g \circ x^{-1})(x(P)) \frac{\partial}{\partial u^i}(f \circ x^{-1})(x(P))$$

$$= f(P) \frac{\partial}{\partial x^i}(P)\, g + g(P) \frac{\partial}{\partial x^i}(P)\, f$$