Math 126, Test 1, February 13, 2013: Name: ______________.

Show steps for partial credit. You are allowed a scientific, non-graphing, calculator. No cell-phones, IPODs etc... can be out during the test. Box your answer for full-credit. Thanks and Enjoy! Each problem worth 11pts hence there are 14(11) = 154pts to earn. Also, your name worth 6pts. So, 160pts is perfect score.

1.) Suppose $F(x) = \frac{1}{2x+3}$ and $G(x) = \sqrt{3+x^2}$. Find the formulas for

a. $(FG)(x) = F(x)G(x) = \boxed{\left(\frac{1}{2x+3}\right)\sqrt{3+x^2}}$

b. $(F \circ G)(x) = F(G(x))$

$= \frac{1}{2G+3} = \boxed{\frac{1}{2\sqrt{3+x^2}+3}}$

2.) Calculate the following indeterminant limit:

$\lim_{x\to 3}\left(\frac{1}{x-3}\left[\frac{1}{x}-\frac{1}{3}\right]\right) = \lim_{x\to 3}\left[\frac{1}{x-3}\left(\frac{3-x}{3x}\right)\right]$

$= \lim_{x\to 3}\left[\frac{-(x-3)}{(x-3)(3x)}\right] = \lim_{x\to 3}\left[\frac{-1}{3x}\right] = \boxed{\frac{-1}{9}.}$

3.) Calculate the following limit:

$\lim_{x\to\infty}\left(\frac{2x^2+3x+12}{7x^2+6x-5}\right) = \lim_{x\to\infty}\left(\frac{2+\underbrace{\cancel{3/x}}_{\to 0}+\underbrace{\cancel{12/x^2}}_{\to 0}}{7+\underbrace{\cancel{6/x}}_{\to 0}-\underbrace{\cancel{5/x^2}}_{\to 0}}\right) = \boxed{\frac{2}{7}.}$

4.) Given the graph below, find the value and limits if they exist, if not write d.n.e or ∞ or $-\infty$ as best fits:

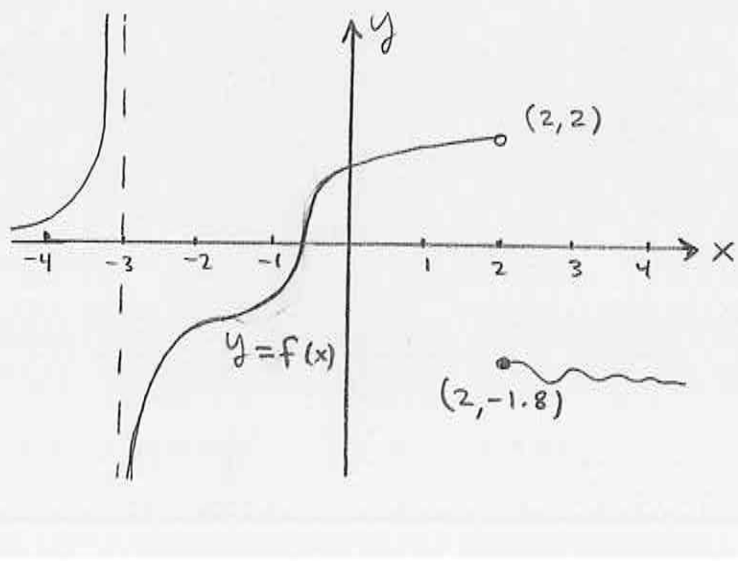
a. $\lim_{x\to 2^+}(f(x)) =$ <u>-1.8</u>.

b. $\lim_{x\to 2^-}(f(x)) =$ <u>2</u>.

c. $\lim_{x\to 2}(f(x)) =$ <u>d.n.e.</u>

d. f(2) = <u>d.n.e.</u>.

e. $\lim_{x\to -3^+}(f(x)) =$ <u>$-\infty$</u>.



not the sol^{n} $\longrightarrow \left(f'(x) = \dfrac{2}{2\sqrt{x+3}} \quad f'(6) = \dfrac{1}{\sqrt{6+3}} = \dfrac{1}{\sqrt{9}} = \dfrac{1}{3} \right)$

5.) Suppose $f(x) = 2\sqrt{x+3}$. Show $f'(6) = \frac{1}{3}$ by the definition of the derivative.
(note: your solution must include explicit limiting arguments for credit)

forbidden here.

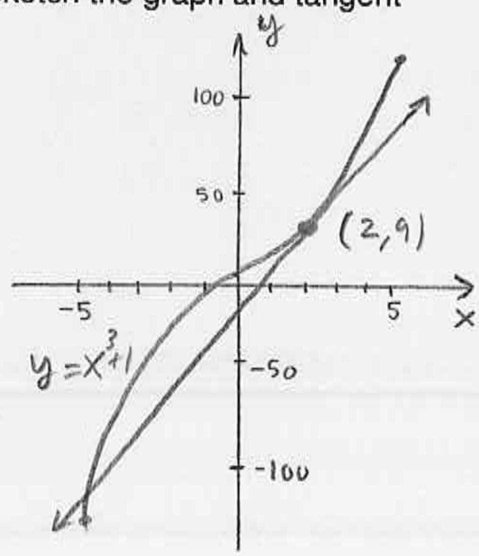
$$f'(6) = \lim_{h\to 0}\left[\frac{f(6+h)-f(6)}{h}\right]$$
$$= \lim_{h\to 0}\left[\frac{2\sqrt{6+h+3} - 2\sqrt{6+3}}{h}\right]$$
$$= \lim_{h\to 0}\left[\left(\frac{2\sqrt{9+h} - 6}{h}\right)\left(\frac{2\sqrt{9+h}+6}{2\sqrt{9+h}+6}\right)\right]$$
$$= \lim_{h\to 0}\left[\frac{(2\sqrt{9+h})^2 + \cancel{2(6)\sqrt{9+h}} - \cancel{2(6)\sqrt{9+h}} - 36}{h(2\sqrt{9+h}+6)}\right]$$
$$= \lim_{h\to 0}\left[\frac{\cancel{36}+4h-\cancel{36}}{h(2\sqrt{9+h}+6)}\right] = \lim_{h\to 0}\left[\frac{4}{2\sqrt{9+h}+6}\right] = \frac{4}{6+6} = \boxed{\frac{1}{3}.}$$

6.) Suppose $f(x) = 2x^2 + 3$. Show $f'(a) = 4a$ by the definition of the derivative.
(note: your solution must include explicit limiting arguments for credit)

$$f'(a) = \lim_{x\to a}\left(\frac{f(x)-f(a)}{x-a}\right) = \lim_{x\to a}\left(\frac{2x^2+3-(2a^2+3)}{x-a}\right)$$
$$= \lim_{x\to a}\left(\frac{2(x^2-a^2)}{x-a}\right)$$
$$= \lim_{x\to a}\left(\frac{2\cancel{(x-a)}(x+a)}{\cancel{x-a}}\right)$$
$$= 2(a+a)$$
$$= \boxed{4a.}$$

7.) If $f(x) = x^3 + 1$ then find the equation of the tangent line at $x = 2$. Sketch the graph and tangent line.

$$f'(x) = 3x^2$$
$$f'(2) = 3(2)^2 = 12.$$
$$\Longrightarrow \quad y = f(2) + f'(2)(x-2)$$
$$\underbrace{\boxed{y = 9 + 12(x-2)}}_{\text{eq}^{n}\text{ of tangent line}}$$



$$f(5) = 125+1 = 126, \quad f(-5) = -124$$

8.) Calculate the derivative below and simplify your answer:

$$\frac{d}{dx}\left[\sqrt[3]{8x^4} + 1/x^2\right] = \frac{d}{dx}\left[\sqrt[3]{8}\,\sqrt[3]{x^4} + x^{-2}\right]$$

$$= \frac{d}{dx}\left[2x^{4/3} - x^{-2}\right] = \boxed{\frac{8}{3}x^{1/3} + 2x^{-3}}$$

Calculate the derivative as indicated in each of the problems that follows. No need to simplify answer, just perform the differentiation and neatly box the result.

9.) $\frac{d}{dx}\left[\underbrace{(2x^2+1)}_{f}\underbrace{\left(x^2 - \frac{1}{x}\right)}_{g}\right] = f'g + fg'$

$$= \boxed{4x\left(x^2 - \frac{1}{x}\right) + (2x^2+1)\left(2x + \frac{1}{x^2}\right)}$$

Chain rule twice.

10.) $\frac{d}{dt}\left[\underbrace{t^{10}}_{f}\underbrace{(2t+10)^3}_{g}\right] = f'g + fg'$

$$= \boxed{10t^9(2t+10)^3 + t^{10}\left(3(2t+10)^2(2)\right)}$$

11.) $\frac{d}{dx}\underbrace{\left(\frac{x+6}{x-6}\right)}_{u}^5 = \frac{d}{dx}(u^5) = 5u^4\frac{du}{dx} = 5u^4\left(\frac{d}{dx}\left(\frac{x+6}{x-6}\right)\right)$

$$= \boxed{5\left(\frac{x+6}{x-6}\right)^4\left[\frac{1(x-6) - 1(x+6)}{(x-6)^2}\right]}$$

$$= 5\left(\frac{x+6}{x-6}\right)^4\left[\frac{-12}{(x-6)^2}\right]$$

$$= \boxed{\frac{-60(x+6)^4}{(x-6)^6}}$$ neat.

12.) $\frac{d}{dx}\sqrt{\underbrace{1+\frac{x}{2x+3}}_{u}} = \frac{d}{dx}\left(\sqrt{u}\right) = \frac{1}{2\sqrt{u}}\frac{du}{dx}$

$$= \frac{1}{2\sqrt{u}}\frac{d}{dx}\left[\cancel{1}^{0} + \overbrace{\frac{x}{2x+3}}\right]$$

quotient, can use quotient rule

$$= \frac{1}{2\sqrt{u}}\left[\frac{1(2x+3)-x(2)}{(2x+3)^2}\right]$$

$$= \boxed{\frac{1}{2\sqrt{1+\frac{x}{2x+3}}}\left[\frac{2x+3-2x}{(2x+3)^2}\right]}$$

$$= \frac{3}{2(2x+3)^2\sqrt{1+\frac{x}{2x+3}}}$$

neat.

13.) Find the interval(s) of real numbers which solve $x^4 - 4x^2 < 0$. Please use a sign-chart to guide your solution.

$$x^2(x^2-4) = x^2(x+2)(x-2) < 0$$

$+++$ | $---$ | $---$ | $+++$ → x^4-4x^2

-2 $\quad 0 \quad$ 2

$\boxed{\text{thus } x \text{ in the intervals } (-2,0) \text{ or } (0,2) \text{ solve the inequality } x^4-4x^2<0}$

14.) On which interval(s) is the function $f(x) = 3x^5 - 20x^3$ increasing ?

$$f'(x) = 15x^4 - 60x^2$$
$$= 15(x^4-4x^2)$$

$y=f(x)$ is increasing when $f'(x) > 0$ but from the previous problem we see $15(x^4-4x^2) > 0$ $\Rightarrow x^4-4x^2 > 0$ hence

$$\boxed{(-\infty,-2) \text{ or } (2,\infty)}$$