Math 126, Test 2, Name:

Show your work, box answers. You are free to use a non-graphical calculator. You can borrow mine for a couple minutes if you forgot your calculator. Each problem is worth 20pts. Thanks!

1. Suppose $R(x) = 600x - 30x^2$ is the revenue function of a business. Find the marginal revenue and determine the value of x which maximizes the revenue.

$$\frac{dR}{dx} = \boxed{600 - 60x} \rightsquigarrow \text{marginal revenue.}$$

$$\text{max: } R'(x) = 0 \Rightarrow 600 = 60x \Rightarrow \boxed{x = 10}$$

2. Find $g''(t)$ given that $g(t) = \sqrt{5t + 2}$

$$g'(t) = \frac{5}{2\sqrt{5t+2}} = \frac{5}{2}(5t+2)^{-1/2}$$

$$g''(t) = \frac{5}{2}\left(\frac{-1}{2}\right)(5t+2)^{-3/2}(5)$$

$$= \boxed{\frac{-25}{4}(5t+2)^{-3/2}}$$

3. Find $f'''(x)$ given that $f(x) = \frac{1}{x} + 4x^3$

$$f'(x) = \frac{-1}{x^2} + 12x^2$$

$$f''(x) = \frac{2}{x^3} + 24x$$

$$\boxed{f'''(x) = \frac{-6}{x^4} + 24}$$

4. Find $\frac{dy}{dx}$ by implicit differentiation of $8x^2 - y^2 = 25y$

$$16x - 2y\frac{dy}{dx} = 25\frac{dy}{dx} \Rightarrow 16x = (25+2y)\frac{dy}{dx}$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{16x}{25+2y}}$$

5. Find an equation of the tangent line to the graph of the equation $(x+y)^3 = y + 26$ at (2,1).

$$3(x+y)^2\left[1+\frac{dy}{dx}\right] = \frac{dy}{dx}$$

$$3(x+y)^2 + \left[3(x+y)^2 - 1\right]\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-3(x+y)^2}{3(x+y)^2-1}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-3(3)^2}{3(3)^2-1} = \frac{-27}{27-1} = \frac{-27}{26}$$

$$\therefore \boxed{y = 1 - \frac{27}{26}(x-2)}$$

6. Suppose Jacob's car travels North at 80mph and Edward's car travels due East 100mph. If they both leave an intersection at the same time then how fast is the distance between them increasing after 30 minutes ?

$$x^2 + y^2 = s^2$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$x = 50$
$y = 40$
$s = \sqrt{50^2+40^2}$

$\frac{dx}{dt} = 100\text{ mph}$

$\frac{dy}{dt} = 80\text{mph}$

$$\frac{ds}{dt} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2s} = \frac{50(100)+40(80)}{\sqrt{50^2+40^2}}$$

$$\boxed{\frac{ds}{dt} = 128.1\text{mph}}$$

7. Find the differential of $f(x) = (3x^2 + 7)^6$.

$$df = 6(3x^2+7)^5(6x)\,dx$$

$$\boxed{df = 36x\,(3x^2+7)^5\,dx}$$

8. The volume of a spherical cancerous tumor is given by the following equation: $V(r) = \frac{4\pi r^3}{3}$. If the radius of a tumor is estimated at 1.2 cm, with a maximum error in measurement of 0.006 cm, determine the error that might occur when the volume of the tumor is calculated.

$$dV = \frac{4}{3}\pi\,(3r^2)\,dr = 4\pi r^2\,dr$$

$$\Rightarrow dV = 4\pi\,(1.2)^2(0.006)\ cm^3 \approx \boxed{0.109\ cm^3}$$

$$V = \frac{4}{3}\pi(1.2)^3 = 7.238\ cm^3$$

$$(\ V \approx 7.238\ cm^3 \pm 0.109\ cm^3\ )$$

9. Find the horizontal and vertical asymptotes of $y = \frac{3x^2+4}{4x^2+x}$. If there is no asymptote of a particular type then state that fact.

$$y = \frac{3x^2+4}{x(4x+1)} \leftarrow \text{zero for } \boxed{\underbrace{x=0}_{V.A.} \text{ or } \underbrace{x=-1/4}_{V.A.}.}$$

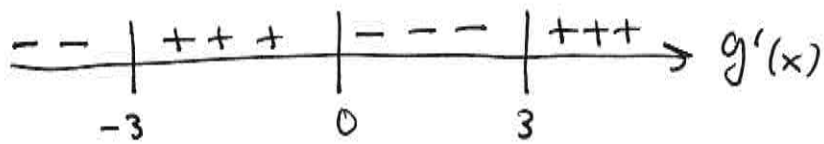
(not a hole in graph since there is no cancelling factor up-top)

$$\lim_{x\to\pm\infty}\left(\frac{3x^2+4}{4x^2+x}\right) = \lim_{x\to\pm\infty}\left(\frac{3+4/x^2}{4+1/x}\right) = \boxed{\frac{3}{4}}$$

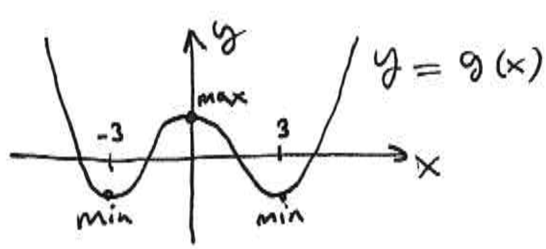
$$\boxed{\text{Horizontal Asymptote of } y = \frac{3}{4}}$$

10. Find the interval(s) where the function $g(x) = x^4 - 18x^2 + 5$ is increasing and the interval(s) where it is decreasing. In addition, determine the location of each local maximum or minimum. Sketch the graph given all this information.

$$g'(x) = 4x^3 - 36x = 4x(x^2-9) = 4x(x+3)(x-3)$$

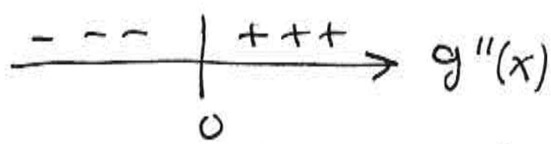


g increasing on $(-3, 0)$ and $(3, \infty)$
g decreasing on $(-\infty, -3)$ and $(0, 3)$
By 1^{st} der. test local max at $x = 0$.
local min at $x = -3, 3$.



11. Determine where $g(x) = 5x^3 - 10x$ is concave up and where it is concave down. Put your answer in interval notation. Is there any point of inflection? If so, where is the point of inflection?

$g'(x) = 15x^2 - 10$
$g''(x) = 30x$



Hence $g(x)$ is concave down on $(-\infty, 0)$
$g(x)$ is concave up on $(0, \infty)$

$(0, g(0)) = (0, 0)$ is a pt. of inflection because we change concavity at that point.