Math 200: Review for Test 2

As always, homework and lecture examples are important. In addition you need to memorize the following defintions.

Be able to state definitions for:

1. definitions from Test 1

   new definitions for Test 2 from Chapter 4

2. function,
3. domain,
4. codomain,
5. range of a function,
6. image,
7. pre-image,
8. image of a set,
9. inverse image of a set,
10. composite function,
11. identity function,

12. inverse function,

13. fiber,

14. restriction of a function,

15. extension of a function,

16. injective (1-1) function,

17. surjective (onto) function,

18. bijection (one-one correspondence),

start of Chapter 5 definitions

19. $\mathbb{N}_k$ for $k \in \mathbb{N}$,

20. equivalence of sets,

21. finite set, cardinality of finite set,

22. infinite set,

23. cardinality $\aleph_o$ and cardnality c (what are these?)

24. countable sets,

25. uncountable sets,

26. countably infinite (denumerable) sets,

start of modular arithmetic defintions, take $n \in \mathbb{N}$ for all the items listed below, this is my Chapter 6 material

27. congruence modulo n, (R_n in my notes to start with, but beware the many notations)

28. equivalence classes of the congruence mod n, (again beware of the notations, make sure to know at least one good notation you can use for your own proofs)

29. addition and multiplication in $\mathbb{Z}_n$ (in terms of $\mathbb{Z}$ and the equivalence classes)

30. multiplicative inverse in $\mathbb{Z}_n$,

31. zero divisors in $\mathbb{Z}_n$,

32. relatively prime,

33. greatest common divisor,

Be able to:

1. prove something is a function

2. prove a function is injective

3. prove a function is surjective

4. prove a function is bijective

5. prove when two functions are equal or show that they are not equal by an example

6. prove relations are functions or show by example that a relation is not a function

7. know and apply Cantor's Theorem

8. be aware of the Theorems from Chapter 4, I don't expect you memorize them, but I might state a few of them on the test and let you apply them to a problem. (or prove them) There are a number of Theorems in Chapter 4 which I provided proofs of in my notes but we didn't cover them explicitly in lecture. Some of those would make good test questions. The proofs really just test your understanding of the definition of function, 1-1 and onto.

9. Be able to graph functions and their restrictions for the standard list of known functions (polynomial, rational, trigonmetric, exponential, logarithmic and piecewise-defined functions)

10. use Proof by Mathematical Induction (PMI). You guys need another question on this. I'll make 15% of the test about this (it's in the take-home portion).

11. prove a function is **single-valued**. This actually goes into proving "something is a function" but I put it separetly here for emphasis.

12. Be able to state the division algorithm (I will not ask you to prove it)

13. Be able to apply the Euclidean algorithm to calculate $gcd(a, b)$ for $a, b \in \mathbb{N}$ (I think it works for $a, b \in \mathbb{Z}$ but I haven't proved that)

14. Be able to find multiplicative inverses of elements in $\mathbb{Z}_n$ for large n. Recall the Euclidean algorithm provided us a new trick beyond guessing, we just work the algorithm backwards to find the inverse. See Example 6.37 for this idea worked out in detail.

15. Be able to prove that $x \in \mathbb{Z}_p$ has a multiplicative inverse in $\mathbb{Z}_p$ if $gcd(x, p) = 1$. (Hint: use the Euclidean Algorithm)

16. Be able to prove that congruence modulo n forms an equivalence relation on $\mathbb{Z}$.

<u>Likely test format:</u>

- 1. [11pts] State from memory the definitions and give an example and counter example of the items requested below.

- 2. [8pts] A collection of true/false, give counter-example questions. The Chapter 5 material will appear here. Mainly I want you to know the definitions of finite, infinite, denumerable and of course set-equivalence.

- 3 . [8pts] Given a function find its range, codomain and fibers. Find a restriction of the function which makes the function injective. Find a reduction of the codomain which makes the reduced function surjective.

- 4. [8pts] Prove a particular relation is a function

- 5. [8pts] Prove a function is onto

- 6. [8pts] Prove a function is a bijection

- 7. [8pts] Find the multiplicative inverse for $x \in \mathbb{Z}_n$ for a particular x and a large value of n (guessing will not work)

- 8. [8pts] Proof of Theorem about onto, or 1-1, composites or perhaps a restriction/extension. There were a number of such proofs in Chapter 4.

- 9. [8pts] Fun with $\mathbb{Z}_n$ problem, perhaps an equation to solve, something from my Chapter 5.

- 10. [10pts (**take-home**)] Questions of a proofy-nature about $\mathbb{Z}_n$. Things like prove $x \in \bar{x}$, or $\bar{x} = \bar{y}$ iff $x \equiv y$ etc...
- 11. [15pts (**take-home**)] Standard PMI problem. Not trivial, but not the binomial Theorem either.

You can expect some of the problems will test understanding of definitions. Other questions will test your ability to construct proofs. The proofs on the test should not involve terribly deep thinking. It will be mostly about your ability to follow a particular method of proof and apply definitions. The take-home problem might involve something less obvious.