Please put your work on these sheets. If you need additional room to show your work then add paper as needed, but be sure to put your answer clearly near the problem statement. Box your answers. Make sure you name is on each page and the assignment is stapled. Thanks and enjoy. This is worth 20pts.

Problem 1 Solve,

(a.) $\frac{dy}{dx} = x^2e^{-4x} - 4y$.

(b.) $\frac{dy}{dx} = \frac{y^2-1}{x^2-1}$ given $y(2) = 2$.

(c.) $(e^x + y)dx + (2 + x + ye^y)dy = 0$ given $y(0) = 1$.

(d.) $\cos(\theta)dr - (r\sin(\theta) - e^{\theta})d\theta = 0$.

Problem 2 Find a continuous solution to the following IVP,

$$\frac{dy}{dx}+2y = f(x), \;\; y(0) = 0, \;\; \text{with} \qquad f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ -1, & x > 1 \end{cases}$$

Problem 3 Show the following differential equation is **not** exact:

$$(5x^2y + 6x^3y^2 + 4xy^2)dx + (2x^3 + 3x^4y + 3x^2y)dy = 0.$$

Solve this ODE by the integrating factor method. Hint: $I = x^Ay^B$ will work here.

Problem 4 Use the substitution $y = vx^2$ to solve

$$\frac{dy}{dx} = \frac{2y}{x} + \cos(y/x^2).$$

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Problem 5 Solve the following by one of the substitution methods we discussed:

(a.) $\frac{dy}{dx} - y = e^{2x}y^3$.

(b.) $\frac{dy}{dx} = (x + y + 2)^2$.

(c.) $m\dot{v} = mg - kv^2$ where m, g, k are constants. Also, find terminal velocity.

(d.) $(x^2 - 2y^2)dx + 2xydy = 0$. Hint: divide by x^2, tilt head, think.

Problem 6 Suppose f is a function such that $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$. Show that either f is the function which is identically zero on $\mathbb{R}$ or f is an exponential function.

Problem 7 Let $\star$ be the DEqn $y^2 \sin(x)dx + yf(x)dy = 0$. Find all functions f such that $\star$ is an exact DEqn.

Problem 8 The DEqn $y = xy' + f(y')$ is called **Clairaut's equation** .

1. Show that the lines $y = cx + f(c)$ are solutions of **Clairaut's equation**
2. Suppose $f(y') = \frac{1}{2}(y')^2$ and show $y = xy' + f(y')$ has solution $y = -\frac{1}{2}x^2$.
3. Plot the so-called **singular** solution $y = -\frac{1}{2}x^2$ and plot the linear solutions at $(-2, -2)$, $(-1, -1/2)$, $(0, 0)$, $(1, -1/2)$ and $(2, -2)$. Clearly Clairaut's equation does not have a unique solution at each point. Does this contradict the uniqueness theorem we discussed in lecture? (why not!)