Physics 231: Test 2: Name: ____________

Show your work. **Box your answers (no box is a 3pt deduction)**. No graphing calculators or other electronic communication devices allowed. There are at least 150pts to earn here. Answers must be given proper units and vector notation where appropriate. Thanks and enjoy! Assume $g = 9.8\ m/s^2$ throughout this test.

[Problem 1][25pts] An essentially massless spring is attached to a 2.0 kg mass and it is allowed to oscillate without friction in a horizontal one-dimensional motion. Suppose the stiffness of the spring is $k = 10\frac{N}{m}$. If at t=0 the mass has a speed $1.2\frac{m}{s}$ as the spring is stretching out at a distance of $0.20\ m$ from its equilibrium then **what is the maximum distance the spring stretches from equilibrium?**

$$E_0 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(2kg)(1.2\frac{m}{s})^2 + \frac{1}{2}(10\frac{N}{m})(0.2m)^2 = 1.64\ J$$

$$E_{max\ dist.} = \frac{1}{2}kX_{max}^2 = 1.64J \hookrightarrow X_{max} = \sqrt{\frac{2(1.64J)}{10N/m}}$$

$$\boxed{X_{max} = 0.5727m}$$

[Problem 2][10pts] An 82 kg running back moving at $5.0\frac{m}{s}$ collides with a 110 kg linebacker who is initially at rest. **What is the resulting velocity of the football players** (assume they stick together after the collision)

$$P_i = (82kg)(5\frac{m}{s}) = (82kg + 110kg)V_f = P_f$$

$$V_f = \left(\frac{(82)(5)}{192}\right)\frac{m}{s} = \boxed{2.135\ m/s}$$

[Problem 3] [15pts] A 7.0 kg box is raised from rest a distance of 2.0 m by a vertical force of 130 N.

rest → $KE_i = 0$

a.) What is the work done by the force?

$$W_F = \vec{F}\cdot\vec{d} = (130N)(2.0m) = \boxed{260J}$$

b.) What is the work done by gravity?

$$W_g = \vec{F}_g\cdot\vec{d} = -mgd = -(7.0kg)(9.8\frac{m}{s^2})(2.0m)$$
$$= \boxed{-137.2J}$$

c.) What is the final kinetic energy of the box?

$$\Delta KE = KE_f - KE_i^{\nearrow 0} = W_{net} = 260J - 137.2J$$

$$\frac{1}{2}mv_f^2 = \boxed{122.8J}$$

$$\left(\text{Btw, } V_f = \sqrt{\frac{245.6J}{7.0kg}} = 5.923\frac{m}{s}\right) \leftarrow \text{not req}^{d}$$

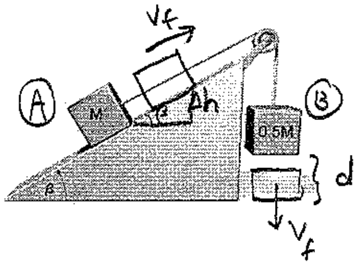
[Problem 4] [10pts] Find the potential energy function for the force $\vec{\mathbf{F}}(\mathbf{x},\mathbf{y}) = \langle 3\mathbf{x}^2+1,\ 2\mathbf{y}\rangle$. Then use your potential energy function to find the work done by the force on a path from (1,1) to (3,4). (formulas in m, kg, s)

$\boxed{U(x,y) = -x^3 - x - y^2}$ has $-\frac{\partial U}{\partial x} = 3x^2+1$ & $-\frac{\partial U}{\partial y} = -2y$

Hence $\vec{F} = -\nabla U$ as desired.

$$W = \int_{(1,1)}^{(3,4)} \vec{F}\cdot d\vec{r} = -\int_{(1,1)}^{(3,4)} \nabla U \cdot d\vec{r} = U(1,1) - U(3,4)$$
$$= (-1-1-1) - (-27-3-16) \Rightarrow \boxed{43\ J}$$

[Problem 5] [25pts] Suppose the masses picture below are initially at rest on the frictionless inclined plane. In addition, suppose $\beta = 30^o$. **What is the speed of the 0.5M mass falling after it has fallen 50cm?**



$\Delta h_M = d\sin\beta$ (up) : $h_{Af} - h_{Ai} = d\sin\beta$

$\Delta h_{0.5M} = -d$ (down) : $h_{Bi} - h_{Bf} = d$

$$E_i = E_f$$
$$Mgh_{Ai} + 0.5Mgh_{Bi} = Mgh_{Af} + 0.5Mgh_{Bf} + KE_f$$
$$KE_f = \frac{1}{2}(M+0.5M)V_f^2 = 0.5Mg(h_{Bi}-h_{Bf}) - Mg(h_{Af}-h_{Ai})$$
$$0.75\,V_f^2 = 0.5g(d) - g(d\sin\beta) \Rightarrow \boxed{V_f = 0\ m/s}$$

[Problem 6] [15pts] A 5.0 kg particle starts from rest at $x = 0$ and moves under the influence of a net-force $F(x) = 6 + 4x + 3x^2$ where we are working in implicit units of kg, m and seconds. **Find the work done by the force as the mass moves from x=0 to x=1.**

Special case of one-dim'l motion.

$$W = \int_C \vec{F}\cdot d\vec{r} = \int_0^1 F(x)dx = \int_0^1 (6+4x+3x^2)dx$$
$$= (6x + 2x^2 + x^3)\Big|_0^1$$
$$= 6 + 2 + 1 = 9 \hookrightarrow \boxed{9\ J}$$

[Problem 7] [15pts] You kick a soccer ball with a mass 0.47 kg. The ball leaves your foot with an initial speed of $29\frac{m}{s}$. If the collision of the ball with your foot takes $0.009\ s$ then what is the magnitude of the average force of your foot on the ball?

$$\Delta P = P_f - P_i = (0.47kg)(29\tfrac{m}{s}) - 0 = 13.63\,\frac{kg\,m}{s}$$

$$F_{avg} = \frac{\Delta P}{\Delta t} = \frac{13.63\ kg\,m/s}{0.009\,s} = \boxed{1514.4\ N}$$

[Problem 8][15pts] A mass $m_1 = 1.0\ kg$ is at (1,2)m. A mass $m_2 = 2.0\ kg$ is at (-1,-2)m. A mass $m_3 = 3.0\ kg$ is at (-1,-2)m. **Find the center of mass for this system.**

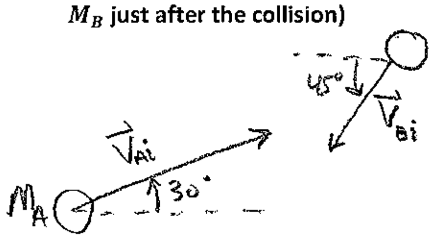
$$M = m_1 + m_2 + m_3 = 6\ kg$$

$$\vec{r}_{cm} = \frac{1}{M}\left(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3\right)$$

$$= \frac{1}{6}\left(\langle 1,2\rangle + 2\langle -1,-2\rangle + 3\langle -1,-2\rangle\right) m$$

$$= \frac{1}{6}\langle 1-2-3,\ 2-4-6\rangle m = \boxed{\langle -2/3,\ -4/3\rangle\ m}$$

[Problem 9] [35pts] A mass $M_A = 3\ kg$ has initial speed of $10\frac{m}{s}$ as it travels in the direction 30 degrees North of East. A second mass $M_B = 10\ kg$ has initial speed of $6\frac{m}{s}$ as it travels in the direction 45 degrees South of West. These masses collide and after the collision M_A bounces off at speed $8\frac{m}{s}$ due north. **What is the speed and direction of M_B just after the collision? (give the direction by stating the standard angle for the velocity vector of M_B just after the collision)**



$$\vec{P}_i = m_A\vec{V}_{Ai} + m_B\vec{V}_{Bi} \qquad \frac{kg\,m}{s}$$

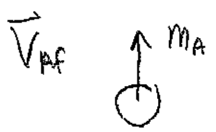
$$= (3)(10)\langle \cos 30, \sin 30\rangle + (10)(6)\langle -\cos 45, -\sin 45\rangle$$

$$= \langle 25.981 - 42.426,\ 15 - 42.426\rangle \frac{kg\,m}{s}$$

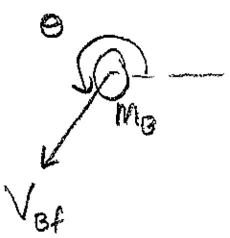
$$= \langle -16.445,\ -27.426\rangle \frac{kg\,m}{s}$$

initial momentum of the system.

~~BOUNCE//~~



$\vec{V}_{Af} = \langle 0, 8\frac{m}{s}\rangle$



$$\vec{P}_f = m_A\vec{V}_{Af} + m_B\vec{V}_{Bf} \quad (\vec{P}_{Bf})$$

(V_{Bf} & θ are unknown)

Conservation of momentum yields $\vec{P}_i = \vec{P}_f$ hence,

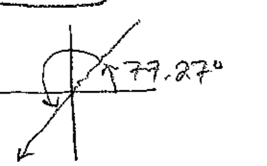
$$\langle -16.445, -27.426\rangle \frac{kg\,m}{s} = \langle 0, 24\frac{kg\,m}{s}\rangle + \vec{P}_{Bf}$$

$$\vec{P}_{Bf} = \langle -16.445,\ -51.426\rangle \frac{kg\,m}{s} = m_B\vec{V}_{Bf}, \quad m_B = 10\,kg$$

$$\vec{V}_{Bf} = \langle -1.6445,\ -5.1426\rangle\ m/s$$

$$\text{speed} = \boxed{V_{Bf} = 5.399\ m/s}$$

$$\tan^{-1}\left(\frac{-5.1426}{-1.6445}\right) = 72.27°$$



$$\boxed{\theta = 252.3°}$$