Physics 231: Test 3: Name:

Show your work. **Box your answers (no box is a 3pt deduction).** No graphing calculators or other electronic communication devices allowed. There are at least 150pts to earn here. Answers must be given proper units and vector notation where appropriate. Thanks and enjoy!

[Problem 1][15pts] A wheel rotates through 5.1 rad in 1.8 s as it is brought to rest with constant angular acceleration. Determine the wheel's initial angular speed before braking began.

$\Delta\theta = 5.1\,\text{rad}, \quad \omega_f = 0\,\text{rad/s}, \quad \alpha = \text{constant}, \quad \Delta t = 1.8\,\text{s}$

$$\omega_f = \omega_0 + \alpha\Delta t \Rightarrow \alpha = -\frac{\omega_0}{\Delta t}$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta \Rightarrow \alpha = -\frac{\omega_0^2}{2\Delta\theta}$$

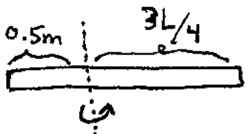
$$\frac{\omega_0}{\Delta t} = \frac{\omega_0^2}{2\Delta\theta}$$

$$\omega_0 = 2\frac{\Delta\theta}{\Delta t} = 2\left(\frac{5.1\,\text{rad}}{1.8\,\text{s}}\right) \cong \boxed{5.67\,\frac{\text{rad}}{\text{s}}}$$

(alternatively, $\omega_0 = 2\omega_{avg}$ ω_0 ω t)

[Problem 2][20pts] The moment of inertia for a rod mass M of length L about one of its ends is $\frac{1}{3}ML^2$. Suppose a rod of mass $M = 2.0\,kg$ of length 2.0m rotates about a point 0.5m from one of its endpoints. If the rotational energy of the rod is $100\,J$ then what is the angular velocity of the rod?

$\frac{7ML^2}{48}$

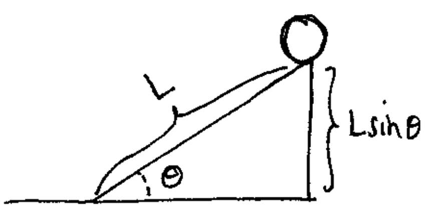


$$I = \frac{1}{3}\left(\frac{M}{4}\right)\left(\frac{L}{4}\right)^2 + \frac{1}{3}\left(\frac{3M}{4}\right)\left(\frac{3L}{4}\right)^2 = \frac{1}{3}M\left(\frac{1}{64} + \frac{27}{64}\right)L^2 = \frac{28ML^2}{3(64)}$$

$$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(\quad ML^2)\omega^2 = 100\,J$$

$$\omega = \sqrt{\frac{2(100\,J)}{I}} = \sqrt{\frac{200\,J}{\frac{7}{48}(2.0\,kg)(2.0\,m)^2}} = \boxed{13.09\,\frac{\text{rad}}{\text{s}}}$$

[Problem 3] [15pts] A solid sphere has moment of inertia $I = \frac{9}{16}MR^2$. If this sphere rolls without slipping down an inclined plane of length L and inclination angle θ then find the speed as it reaches the base of the plane as a function of both L and θ.



$$E = mgy + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2, \quad \omega = \frac{v}{R}$$

$$E = mgy + \frac{1}{2}mv^2 + \frac{9}{32}mv^2$$

$$E = mgy + \frac{25}{32}mv^2$$

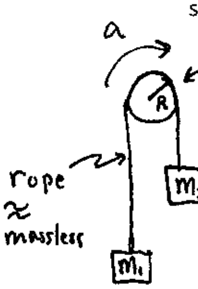
$$E_0 = \cancel{m}gL\sin\theta = \frac{25}{32}\cancel{m}v_f^2 = E_f$$

$$\boxed{v_f = \sqrt{\frac{32gL\sin\theta}{25}}} = \frac{2}{5}\sqrt{8gL\sin\theta} = \frac{4}{5}\sqrt{2gL\sin\theta}$$

[Problem 4][10pts] A net-force of $\vec{F} = < 10, 0, -3 > N$ is applied to a solid body at the point (1,2,3)m. Find the torque on the body with respect to the origin (0,0,0)m.

$$\vec{\tau} = \vec{r} \times \vec{F} = \langle 10, 0, -3 \rangle$$
$$= \langle 1, 2, 3 \rangle \times \langle 10, 0, -3 \rangle \text{ Nm}$$
$$= \langle -6, 30+3, -20 \rangle \text{Nm} = \boxed{\langle -6, 33, -20 \rangle \text{Nm}}$$

[Problem 5] [25pts] A mass $M_1 = 10\ kg$ hangs off the left of a pulley with moment of inertia $I = 2.0\ kg\ m^2$ and radius $R = 8.0\ cm$. A second mass $M_2 = 30\ kg$ hangs off the right of the pulley. What is the acceleration of the system? What is the tension T_1 in the rope where M_1 hangs?



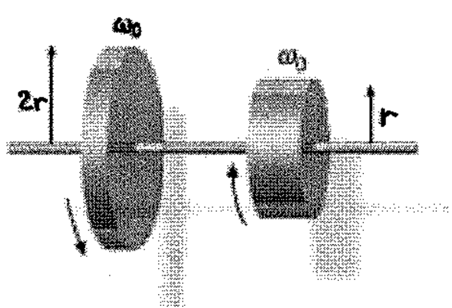
$$\left.\begin{aligned} m_1 a &= -m_1 g + T_1 \\ m_2 a &= m_2 g - T_2 \end{aligned}\right\} \Rightarrow T_1 - T_2 = (m_1+m_2)a + (m_1 - m_2)g \quad \star$$

$$I\alpha = RT_2 - RT_1 \Rightarrow T_2 - T_1 = \frac{Ia}{R^2} \quad \star\star \qquad \text{since } \alpha = \frac{a}{R}$$

Equate $\star = -\star\star$ to obtain

$$(m_1+m_2)a + (m_1 - m_2)g = \frac{-Ia}{R^2}$$
$$\left(m_1 + m_2 + \frac{I}{R^2}\right)a = (m_2 - m_1)g$$
$$a = \left(\frac{m_2 - m_1}{m_1 + m_2 + \frac{I}{R^2}}\right) g = \boxed{0.556 \frac{m}{s^2}}, \quad T_1 = m_1 a + m_1 g = \boxed{103.6\ N}$$

[Problem 6] [15pts] Two disks of identical mass but different radii (r and $2r$) are spinning on frictionless bearings at the same angular speed ω_0, but in opposite directions. The two disks are brought slowly together. The resulting frictional force between the surfaces eventually brings them to a common angular velocity. Find the final angular velocity ω_f.

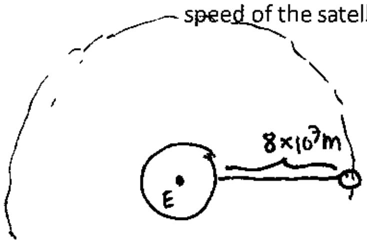


$$L_0 = \frac{1}{2}M(2r)^2\omega_0 - \frac{1}{2}Mr^2\omega_0$$

$$L_f = \left(\frac{1}{2}M(2r)^2 + \frac{1}{2}mr^2\right)\omega_f$$

$$L_0 = L_f \Rightarrow \omega_f = \frac{\frac{3}{2}Mr^2\omega_0}{\frac{5}{2}Mr^2} = \boxed{\frac{3}{5}\omega_0}$$

[Problem 7] [15pts] Note: $M_{earth} = 5.97 \times 10^{24} kg$, $R_{earth} = 6.38 \times 10^{6} m$ and $G = 6.673 \times 10^{-11}\ Nm^2/kg^2$. A satellite with a mass of $m = 270\ kg$ moves in a circular orbit $8.00 \times 10^{7} m$ above the Earth's surface. What is the speed of the satellite?



$$\frac{mv^2}{R} = \frac{G m M_E}{R^2}$$ (circular motion due to earth's gravitational pull)

$$V = \sqrt{\frac{G M_E}{R}} = \sqrt{\frac{(6.673\times10^{-11})(5.97\times10^{24})}{86.38\times10^{6}}}\ \frac{m}{s}$$

$R = (6.38\times10^{6} + 8\times10^{7})m$

$R = 86.38\times10^{6} m$

$$= \boxed{2147.5\ \frac{m}{s}}$$

[Problem 8][15pts] A mass of $2.0\ kg$ is attached to an essentially massless spring which causes the mass to have the equation of motion $x(t) = 10\ sin(6t)$. (in kg, m and s). Find:

$\omega^2 = \frac{k}{m}$ $\quad k = \omega^2 m = 72\ \frac{kg}{s^2}$

(a.) period

$\boxed{T = \frac{2\pi}{6}\ s}$

$= 1.047 s$

(b.) angular frequency

$\boxed{\omega = 6\frac{m}{s}} = \frac{2\pi}{T}$

(c.) total energy for the system.

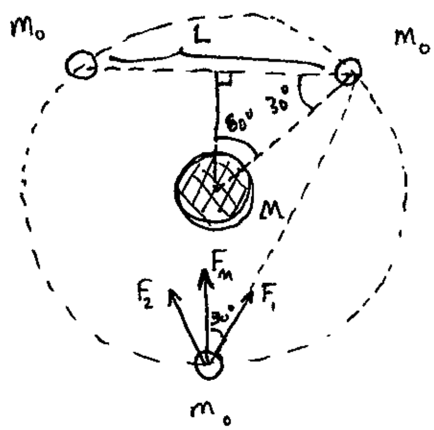
$E = \frac{1}{2}kA^2 = \left(36\ \frac{kg}{s^2}\right)(10m)^2 = \boxed{3600\ J}$

[Problem 9][20pts] Consider binary star system is a pair of stars which orbit a common center. Suppose the stars are identical with mass m_o orbit in a common orbital plane and suppose they orbit in a circle a distance 2L from each other. What is the speed of the stars orbit?

the gravitational pull of the opposite star provides the needed force to maintain the center-seeking centripetal acceleration of $V^2/R = V^2/L$.

$$F = \frac{G m m_o}{(2L)^2} = \frac{m_o V^2}{L} \Rightarrow \boxed{V = \sqrt{\frac{G m_o}{4L}}}$$

[Problem 10][20pts] The Kanagy clan makes its home on a distant planet of mass M with three moons. Suppose the moons are identical with mass m_o orbit in a common orbital plane and suppose they orbit in a circle a distance L from each other. What is the speed of the lunar orbits?

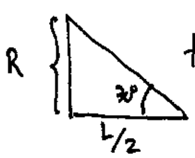


$$F_{net,\ center\ seeking} = \frac{GMm_o}{\underbrace{\left(\frac{L}{2\sqrt{3}}\right)}_{R}^2} + 2\left(\frac{G m_o m_o}{L^2}\right)\overbrace{\cos 30^\circ}^{\frac{\sqrt{3}}{2}} = \frac{m_o V^2}{R}$$

$$V = \sqrt{\frac{GM}{R} + \frac{\cancel{2}RGm_o\sqrt{3}}{\cancel{2}L^2}} \qquad \left(R\sqrt{3} = \frac{L}{2}\right)$$

$$= \sqrt{G\left(\frac{2\sqrt{3}M}{L} + \frac{m_o}{2L}\right)}$$

$$= \boxed{\sqrt{\frac{G}{L}\left(2\sqrt{3}M + \frac{1}{2}m_o\right)}}$$



$\tan 30^\circ = \frac{R}{L/2}$

$$R = \frac{L \tan 30}{2} = \frac{L}{2}\left(\frac{1/2}{\sqrt{3}/2}\right) = \frac{L}{2\sqrt{3}}$$