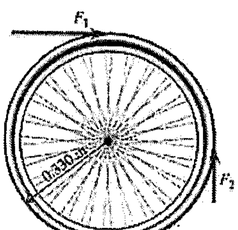
You are allowed a 3" × 5" card. Same instructions as previous test.

Problem 1 (15pts) Let a force $\vec{F} = \langle 0, 6, 3\rangle\, N$ be applied to a particle at position $\vec{r} = \langle 2, 1, 0\rangle\, m$. Find the torque on the particle.

$$\vec{\tau} = \vec{r} \times \vec{F} = \langle 2, 1, 0\rangle \times \langle 0, 6, 3\rangle \text{ Nm}$$
$$= \boxed{\langle 3, -6, 12\rangle \text{ Nm}}$$

Problem 2 (15pts) Forces $F_1 = 7.60\ N$ and $F_2 = 4.30\ N$ are applied tangentially to a wheel with radius $0.330\ m$, as shown below. **Find the net torque on the wheel.**
Also, please **circle one of the following** to indicate the torque's direction:



$$\tau_{net} = RF_2 - RF_1 \qquad (ccw > 0)$$
$$= (0.330m)(4.30N - 7.60N)$$
$$= \boxed{-1.089 \text{ Nm}}$$

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Problem 3 (10pts) A yo-yo has $1000\, J$ of energy in the form of rotational kinetic energy. The yo-yo also has an angular momentum of $L = 20\, m^2 kg/s$. What is the moment of intertia of the yo-yo ?

$$KE = \frac{1}{2} I \omega^2 \quad \text{and} \quad \underbrace{L = I\omega}_{\omega = L/I} \Rightarrow KE = \frac{1}{2} I \left(\frac{L}{I}\right)^2 = \frac{L^2}{2I}$$

$$I = \frac{L^2}{2KE} = \frac{(20\, m^2 kg/s)^2}{2000\, J} = \frac{400}{2000} \frac{(kg^2/s^2)\, m^4}{kg\, m^2/s^2} = \boxed{0.2\, kg\, m^2}$$

Problem 4 (10pts) Suppose you are on a moon where the gravitational acceleration is $4 m/s^2$ when you are $10\, km$ from the center of the moon. What is the mass of this moon ?

$$ma = \frac{GmM}{R^2} \rightarrow a = \frac{GM}{R^2}$$

$$\therefore M = \frac{aR^2}{G} = \frac{(4.0\, m/s^2)(10 \times 10^3 m)^2}{6.673 \times 10^{-11} \frac{Nm^2}{kg^2}} = \boxed{5.994 \times 10^{18} kg}$$

Problem 5 (15pts) Suppose big lump of peanut butter is dropped onto a record which is initially spinning at 45 revolutions per minute. Assume the mass of the peanut butter is 0.3 kg and the mass of the record is 0.8 kg. The peanut butter is dropped at the outer rim of the 22.0 cm diameter record which is of uniform density. If the peanut butter is dropped vertically onto the record then what is the angular velocity (in radians per second) at which the record spins right after the peanut butter sticks to the record.

$$\omega_i = 45\,rpm \qquad m = 0.3\,kg \qquad M = 0.8\,kg \qquad R = 11.0\,cm$$

$$L_i = L_f$$

$$\left(\tfrac{1}{2} M R^2\right)\omega_i = \left(\tfrac{1}{2} M R^2 + m R^2\right)\omega_f$$

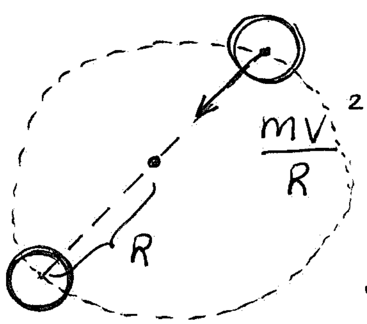
$$\omega_f = \frac{\tfrac{1}{2} M R^2 \omega_i}{\tfrac{1}{2} M R^2 + m R^2} = \left(\frac{M}{M + 2m}\right)\omega_i$$

$$\omega_f = \left(\frac{0.8\,kg}{0.8\,kg + 0.6\,kg}\right)\left(45\,\frac{rev}{min}\cdot\frac{min}{60\,s}\cdot\frac{2\pi\,rad}{rev}\right)$$

$$\boxed{\omega_f = 2.693\,\frac{rad}{s}}$$

Problem 6 (15pts) A pair of moons of mass M orbit due to each other's gravitational attraction If the moons follow a circular orbit in centered around a central point then what is the speed of their orbit as a function of its radius R and the given masses etc.

(oops, $m = M$)

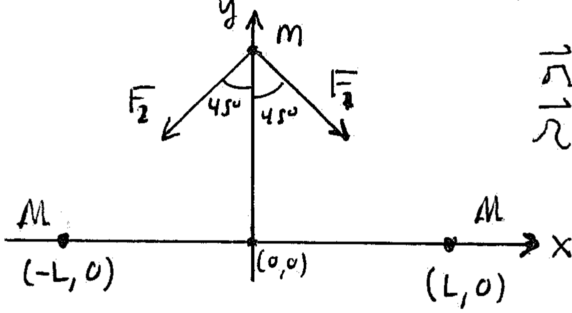


$$\frac{mv^2}{R} = \frac{Gmm}{(2R)^2} = \frac{Gmm}{4R^2}$$

$$v^2 = \frac{Gm}{4R}$$

$$\boxed{v = \sqrt{\frac{Gm}{4R}} = \frac{1}{2}\sqrt{\frac{GM}{R}}}$$

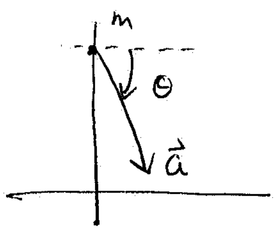
Problem 7 (20pts) Suppose $M_1 = M$ is at $(L, 0)$ and a mass $M_2 = 2M$ is at $(-L, 0)$. Find the magnitude and direction of the gravitational force on m if we place m at the point $(0, L)$. Describe the direction in terms of the **standard angle** for the gravitational force on m.



$$\vec{r}_1 = \langle L, -L \rangle$$
$$\vec{r}_2 = \langle -L, -L \rangle$$
$$r_2 = r_1 = L\sqrt{2}$$

$$m\vec{a} = \vec{F}_1 + \vec{F}_2 = \frac{GmM_2}{r_2^3}\vec{r}_2 + \frac{GmM_1}{r_1^3}\vec{r}_1$$

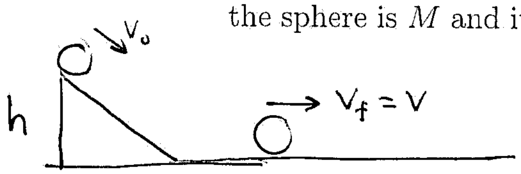
$$\vec{a} = \frac{2GM}{(L\sqrt{2})^3}\langle -L, -L \rangle + \frac{GM}{(L\sqrt{2})^3}\langle L, -L \rangle$$



$$\vec{a} = \frac{GM}{2^{3/2}L^2}\langle -2+1, -2-1 \rangle$$

$$\vec{a} = \frac{GM}{L^2 2^{3/2}}\langle -1, -3 \rangle = \frac{GM}{L^2}\left\langle \frac{-1}{2^{3/2}}, \frac{-3}{2^{3/2}} \right\rangle$$

Problem 8 (20pts) Find the speed at which a solid sphere rolls after it rolls down an inclined plane of height h without slipping given that it is initially pushed with a speed v_o. If the mass of the sphere is M and its radius is R find its final speed v as reaches the base of the incline.



$$V_o = R\omega_o$$
$$I = \frac{2}{5}MR^2$$

$$Mgh + \frac{1}{2}mv_o^2 + \frac{1}{2}I\omega_o^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Now, $\omega = V/R$ and $I = \frac{2}{5}MR^2$

thus $\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2 = \frac{1}{5}MV^2$

Likewise, $\frac{1}{2}I\omega_o^2 = \frac{1}{5}MV_o^2$ hence

$$Mgh + \left(\frac{1}{2} + \frac{1}{5}\right)MV_o^2 = \left(\frac{1}{2} + \frac{1}{5}\right)MV^2$$

$$gh + \frac{7}{10}V_o^2 = \frac{7}{10}V^2$$

$$\therefore \quad V^2 = \frac{10gh}{7} + V_o^2 \implies \boxed{V = \sqrt{V_o^2 + \frac{10gh}{7}}}$$

$$a = \frac{GM}{L^2}\sqrt{\left(\frac{-1}{2^{3/2}}\right)^2 + \left(\frac{-3}{2^{3/2}}\right)^2}$$

$$\underbrace{\boxed{a = \frac{GM}{L^2}\sqrt{\frac{10}{8}} \approx 1.118\frac{GM}{L^2}}}_{\text{magnitude*}}$$

$$\underbrace{\boxed{\theta = 180^\circ + \tan^{-1}(3) = 251.57^\circ}}_{\text{standard angle}}$$

* pop pop.

Problem 9 (20pts) A carousel has an initial angular velocity of 20 revolutions per minute. You are also given that the moment of inertia for the carousel is 200 $m^2 kg$. If the brake for the carousel is applied at a point which is 3.0 m from the axis of rotation with a force of 100 N then **(a.)** find how much time it takes for the brake make the carousel come to rest, and **(b.)** the number of radians the carousel rotates through as it is slowing to a stop.

(a.) $\tau = I\alpha = -RF = (-3.0m)(100N) = -300\ Nm = (200 m^2 kg)\,\alpha$

Therefore $\underline{\alpha = -1.5\ rad/s^2}$. Notice $\omega_o = 20 \frac{rev}{min} \cdot \frac{min}{60s} \cdot \frac{2\pi\ rad}{rev}$

hence $\omega_o = 2.0944\ rad/s$

$$\omega_f^2 = \omega_o + \alpha t = 0 \rightarrow t = \frac{-\omega_o}{\alpha} = \frac{-2.0944\ rad/s}{-1.5\ rad/s^2}$$

$$\Rightarrow \boxed{t \cong 1.396 s}$$

(b.) $\omega_f^2 = \omega_o^2 + 2\alpha\Delta\theta$

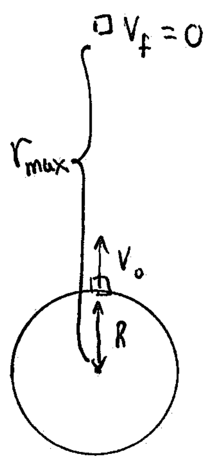
$$\Delta\theta = \frac{-\omega_o^2}{2\alpha} = \frac{-(2.0944\ rad/s)^2}{-3\ rad/s^2} \cong \boxed{1.462\ rad}$$

Problem 10 (20pts) Suppose Pastor Chuck chucks a roll vertically with speed v_o given below. If we assume the roll is thrown from the surface of planet Bojangles with $M = 4.0 \times 10^{24}\ kg$ and a radius of $R = 5000\ km$ then find the maximum height reached by the roll if:

(a.) $v_o = 20\ km/s$

(b.) $v_o = 5\ km/s$

$$PE = \frac{-GmM}{r} = \frac{-GmM}{R+h}$$



$$E = \frac{1}{2}mV_o^2 - \frac{GmM}{R} = -\frac{GmM}{R+h}$$

$$V_o^2 = 2GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\frac{1}{R} - \frac{1}{R+h} = \frac{V_o^2}{2GM}$$

$$\frac{1}{R} - \frac{V_o^2}{2GM} = \frac{1}{R+h} \rightarrow h = \frac{1}{\frac{1}{R} - \frac{V_o^2}{2GM}} - R$$

$r_{max} = R + h$

$$h = R\left(\frac{1}{1 - V_o^2 R/2GM} - 1\right)$$

$$= R\left(\frac{V_o^2 R/2GM}{1 - V_o^2 R/2GM}\right) = \frac{V_o^2 R^2}{2GM - V_o^2 R}$$

(a.) $h < 0 \Rightarrow \boxed{\text{no max h.}}$

(b.) $\boxed{h = 1.529 \times 10^6 m = 1529 km}$