Integrate

a. $\int x\cos(2x^2 - 1)\ dx$

b. $\int \frac{e^{\frac{1}{x}}}{x^2}\ dx$

c. $\int \frac{5\cot(\sqrt[3]{x})}{\sqrt[3]{x^2}}\ dx$

d. $\int \csc(5x+2)\ dx$

e. $\int \frac{e^x}{(3+e^x)^5}\ dx$

f. $\int \sinh x \cosh^2 x\ dx$

g. $\int \frac{5x}{\cos(x^2+2)}\ dx$

h. $\int \frac{x-5}{\sqrt{x+2}}\ dx$

i. $\int 5^x \tan(5^x)\ dx$

j. $\int \frac{\csc(2\ln x)\cot(2\ln x)}{5x}\ dx$

k. $\int \sin(2x)\, 5^{\sin^2 x}\ dx$

l. $\int \frac{\sin x}{\cos^3 x}\ dx$

m. $\int \frac{e^{2x}}{(2+e^x)^2}\ dx$

n. $\int \frac{1}{x^2+a}\ dx \quad a > 0$

Substitution

a) $\int x\cos(2x^2-1)\,dx$

$\boxed{u = 2x^2-1}$

$du = 4x\,dx$

$\boxed{\frac{1}{4}du = x\,dx}$

$= \int \cos u \left(\frac{1}{4}du\right)$

$= \frac{1}{4}\sin u + C$

$= \frac{1}{4}\sin(2x^2-1) + C$

b) $\int \frac{e^{1/x}}{x^2}\,dx$

$\boxed{u = \frac{1}{x}}$

$du = \frac{-1}{x^2}\,dx$

$\boxed{-du = \frac{1}{x^2}\,dx}$

$= \int e^u(-du)$

$= -e^u + C$

$= -e^{1/x} + C$

c) $\int \frac{5\cot(\sqrt[3]{x})}{\sqrt[3]{x^2}}\,dx$

$\boxed{u = \sqrt[3]{x} = x^{1/3}}$

$du = \frac{1}{3}x^{-2/3}\,dx$

$\boxed{3\,du = \frac{1}{\sqrt[3]{x^2}}\,dx}$

$= \int 5\cot u\,(3\,du)$

$= 15\ln|\sin u| + C$

$= 15\ln|\sin(\sqrt[3]{x})| + C$

②

d) $\int \csc(5x+2)\,dx$

$\boxed{u = 5x+2}$

$du = 5\,dx$

$\boxed{\frac{1}{5}\,du = dx}$

$= \int \csc u \left(\frac{1}{5}\,du\right)$

$= \frac{1}{5} \ln|\csc u - \cot u| + C$

$= 1/5 \ln|\csc(5x+2) - \cot(5x+2)| + C$

e) $\int \frac{e^x}{(3+e^x)^5}\,dx$

$\boxed{u = 3+e^x}$

$\boxed{du = e^x\,dx}$

$= \int \frac{1}{u^5}\,du = \int u^{-5}\,du$

$= -\frac{1}{4} u^{-4} + C$

$= -1/4\,(3+e^x)^{-4} + C$

f) $\int \sinh x \cosh^2 x\,dx$

$\boxed{u = \cosh x}$

$\boxed{du = \sinh x\,dx}$

$= \int u^2\,du$

$= 1/3\,u^3 + C$

$= 1/3 \cosh^3 x + C$

g) $\displaystyle\int \frac{5x}{\cos(x^2+2)}\,dx$

$\boxed{u = x^2+2}$

$du = 2x\,dx$

$\boxed{\tfrac{1}{2}\,du = x\,dx}$

$$= \int \frac{5}{\cos u}\left(\tfrac{1}{2}du\right)$$

$$= \tfrac{5}{2}\int \sec u\,du$$

$$= \tfrac{5}{2}\ln|\sec u + \tan u| + C$$

$$= \tfrac{5}{2}\ln|\sec(x^2+2) + \tan(x^2+2)| + C$$

$$= \tfrac{5}{2}\ln|$$

h) $\displaystyle\int \frac{x-5}{\sqrt{x+2}}\,dx$

$u = x+2$

$du = dx$

$x = u-2$

$x-5 = u-7$

$$= \int \frac{u-7}{\sqrt{u}}\,du$$

$$= \int \left(u^{1/2} - 7u^{-1/2}\right)du$$

$$= \tfrac{2}{3}u^{3/2} - 7\left(2u^{1/2}\right) + C$$

$$= \tfrac{2}{3}(x+2)^{3/2} - 14(x+2)^{1/2} + C$$

④

i) $\int 5^x \tan(5^x)\,dx$

$u = 5^x$

$du = (\ln 5)\,5^x\,dx$

$\frac{1}{\ln 5}\,du = 5^x\,dx$

$= \int \tan u \left(\frac{1}{\ln 5}\,du\right)$

$= \frac{1}{\ln 5} \ln|\sec u| + C$

$= \frac{1}{\ln 5} \ln\left|\sec(5^x)\right| + C$

j) $\int \frac{\csc(2\ln x)\cot(2\ln x)}{5x}\,dx$

$u = 2\ln x$

$du = \frac{2}{x}\,dx$

$\frac{1}{2}\,du = \frac{1}{x}\,dx$

$= \int \frac{\csc u \cot u}{5}\left(\frac{1}{2}\,du\right)$

$= -\frac{1}{10}\csc u + C$

$= -\frac{1}{10}\csc(2\ln x) + C$

k $\int \sin(2x)\, 5^{\sin^2 x}\,dx$

$u = \sin^2 x$

$du = 2\sin x\cos x\,dx$

$\boxed{du = \sin(2x)\,dx}$

$= \int 5^u\,du$

$= \frac{1}{\ln 5}\cdot 5^u + C$

$= \frac{1}{\ln 5}\, 5^{\sin^2 x} + C$

l) $\int \frac{\sin x}{\cos^3 x}\,dx$

$u = \cos x$

$du = -\operatorname{sen} x\,dx$

$-du = \operatorname{sen} x\,dx$

$= \int \frac{1}{u^3}\,du = \int u^{-3}\,du$

$= -\frac{1}{2}u^{-2} + C$

$= -\frac{1}{2}\frac{1}{\cos^2 x} + C = -\frac{1}{2}\sec^2 x + C$

m) $\int \frac{e^{2x}}{(2+e^x)^2}\,dx$

$u = 2+e^x$

$du = e^x\,dx$

$e^x = u-2$

$= \int \frac{e^x \cdot e^x}{(2+e^x)^2}\,dx$

$= \int \frac{u-2}{u^2}\,du$

$= \int \left(\frac{1}{u} - 2u^{-2}\right) du$

$= \ln|u| + \frac{2}{u} + C$

$= \ln|2+e^x| + \frac{2}{2+e^x} + C$

$$\int \frac{1}{x^2+a}\,dx \qquad a>0$$

$$= \frac{1}{a}\int \frac{1}{\frac{x^2}{a}+1}\,dx$$

$$u = \frac{x}{\sqrt{a}}$$

$$du = \frac{1}{\sqrt{a}}\,dx$$

$$\sqrt{a}\,du = dx$$

$$= \frac{1}{a}\int \frac{1}{\left(\frac{x}{\sqrt{a}}\right)^2+1}\,dx$$

$$= \frac{1}{a}\int \frac{1}{u^2+1}\left(\sqrt{a}\,du\right)$$

$$= \frac{\sqrt{a}}{a}\tan^{-1}u + C$$

$$= \frac{1}{\sqrt{a}}\tan^{-1}\left(\frac{x}{\sqrt{a}}\right) + C \quad \text{or} \quad \frac{\sqrt{a}}{a}\tan^{-1}\left(\frac{\sqrt{a}}{a}x\right) + C$$

This is a basic form!!