Math 131 Integration Multiple Methods

a. $\displaystyle\int \frac{5\sin^2 x - 2\sin x + 7}{3\tan x}\, dx$

b. $\displaystyle\int \frac{\cos^3 x}{3\sin x}\, dx$

c. $\displaystyle\int \frac{\cos(2x)}{\cos^4 x}\, dx$

d. $\displaystyle\int \left(\frac{\ln^3 x}{x} + \frac{3}{5\sqrt[3]{x}}\right) dx$

e. $\displaystyle\int \left(3^{\sin x}\cos x + \sin x \cos^3 x\right) dx$

f. $\displaystyle\int \frac{\sinh x}{e^x}\, dx$

g. $\displaystyle\int \frac{\sinh x + \cosh x}{e^x}\, dx$

h. $\displaystyle\int \frac{\ln x - 9}{x(\ln^2 x + 2)}\, dx$

More Integration Problems

a) $$\int \frac{5\sin^2 x - 2\sin x + 7}{3\tan x}\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$= \int \frac{(5\sin^2 x - 2\sin x + 7)\cos x}{3\sin x}\,dx$$

$$= \int \frac{5u^2 - 2u + 7}{3u}\,du$$

$$= \int \left(\frac{5}{3}u - \frac{2}{3} + \frac{7}{3}u^{-1}\right)du$$

$$= \frac{5}{3}\left(\frac{1}{2}u^2\right) - \frac{2}{3}u + \frac{7}{3}\ln|u| + C$$

$$= \frac{5}{6}\sin^2 x - \frac{2}{3}\sin x + \frac{7}{3}\ln|\sin x| + C$$

b) $$\int \frac{\cos^3 x}{3\sin x}\,dx$$

$$= \int \frac{\cos x(\cos^2 x)}{3\sin x}\,dx$$

$$= \int \frac{\cos x(1-\sin^2 x)}{3\sin x}\,dx$$

$$= \int \left(\frac{1}{3}\cot x - \frac{1}{3}\sin x\cos x\right)dx$$

2nd part: $u = \sin x$

$du = \cos x\,dx$

$$= \int \frac{1}{3}\cot x\,dx - \int \frac{1}{3}\sin x\cos x\,dx$$

$$= \frac{1}{3}\ln|\sin x| - \frac{1}{3}\int u\,du$$

$$\downarrow \qquad -\frac{1}{3}\left(\frac{1}{2}u^2\right) + C$$

$$-\frac{1}{6}\sin^2 x + C$$

$$= \frac{1}{3}\ln|\sin x| - \frac{1}{6}\sin^2 x + C$$

② c) $\displaystyle\int \frac{\cos(2x)}{\cos^4 x}\,dx$

$$= \int \frac{\cos^2 x - \sin^2 x}{\cos^4 x}\,dx$$

$$= \int \left(\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^4 x}\right) dx$$

$$= \int \left(\sec^2 x - \sec^2 x \tan^2 x\right) dx$$

2nd part: $u = \tan x$
$du = \sec^2 x$

$$= \int \sec^2 x\,dx - \int \sec^2 x \tan^2 x\,dx$$

$$= \tan x - \int u^2\,du$$

$$-\frac{1}{3}u^3 + C$$

$$= \tan x - \frac{1}{3}\tan^3 x + C$$

d) $\displaystyle\int \left(\frac{\ln^3 x}{x} + \frac{3}{5\sqrt[3]{x}}\right) dx$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$$= \int \frac{\ln^3 x}{x}\,dx + \int \frac{3}{5} x^{-1/3}\,dx$$

$$= \int u^3\,du + \frac{3}{5}\left(\frac{3}{2} x^{2/3}\right) + C$$

$$= \frac{1}{4}u^4 + \frac{9}{10} x^{2/3} + C$$

$$= \frac{1}{4}\ln^4 x + \frac{9}{10} x^{2/3} + C$$

③

e) $\int (3^{\sin x} \cos x + \sin x \cos^3 x)\, dx$

$u = \sin x$ | $v = \cos x$

$du = \cos x\, dx$ | $dv = -\sin x\, dx$

$-dv = \sin x\, dx$

$= \int 3^{\sin x} \cos x\, dx + \int \sin x \cos^3 x\, dx$

$= \int 3^u\, du - \int v^3\, dv$

$= \frac{1}{\ln 3} 3^u - \frac{1}{4} v^4 + C$

$= \frac{1}{\ln 3} 3^{\sin x} - \frac{1}{4} \cos^4 x + C$

f. $\int \frac{\sinh x}{e^x}\, dx$

$= \int \frac{e^x - e^{-x}}{e^x}\, dx$

$u = -2x$

$du = -2\, dx$

$-\frac{1}{2} du = dx$

$= \int (1 - e^{-2x})\, dx$

$= x - \int e^{-2x}\, dx$

$= x - \int e^u (-\tfrac{1}{2}\, du)$

$= x + \frac{1}{2} e^u + C$

$= x + \frac{1}{2} e^{-2x} + C$

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g) $\displaystyle\int \frac{\sinh x + \cosh x}{e^x}\,dx$

$$= \int \frac{\frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2}}{e^x}\,dx$$

$$= \int \frac{2e^x}{2e^x}\,dx = \int 1\,dx = x + C$$

h) $\displaystyle\int \frac{\ln x - 9}{x(\ln^2 x + 2)}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}\,dx$

$$= \int \frac{u-9}{u^2+2}\,du$$

$v = u^2+2$

$dv = 2v\,dv$

$\frac{1}{2}\,dv = v\,dv$

$$= \int \frac{u}{u^2+2}\,du - \int \frac{9}{u^2+2}\,du$$

$$= \int \frac{1}{v}\left(\tfrac{1}{2}\,dv\right) - \frac{9}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C$$

$$= \frac{1}{2}\ln|v| - \frac{9}{\sqrt{2}}\tan^{-1}\left(\frac{\ln x}{\sqrt{2}}\right) + C$$

$$= \frac{1}{2}\ln(u^2+2)$$

$$= \frac{1}{2}\ln(\ln^2 x + 2) - \frac{9}{\sqrt{2}}\tan^{-1}\left(\frac{\ln x}{\sqrt{2}}\right) + C$$