Please work the problems in the white space provided and clearly box your solutions. If there is not enough space please write the answer in the given space and indicate that the work is supplied on a separate sheet immediately following the problem. Enjoy!

remember, you can check your work with a CAS, but it is you who must do the work. Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations.

Problem 1 Let A be a positive constant. Integrate. (these are solvable without IBP)

(a)

$$\int_0^{\pi} \sqrt{\sin^2(2t)}\, dt$$

(b)

$$\int x\sqrt{3-4Ax}\, dx$$

(c)

$$\int \frac{dx}{x\ln(x^A)}$$

(d)

$$\int A^x A^{Ax}\, dx$$

(e)

$$\int_{-1}^{1} \frac{\sinh(Ax)dx}{\cos(Ax)}$$

Problem 2 Integrate. (these are solvable without IBP)

(a)

$$\int \cosh^2(x)\,dx$$

(b)

$$\int 2^x \cosh(x)\,dx$$

(c)

$$\int \frac{dx}{1-x^2}$$

Problem 3 Let a be a positive constant. Use integration by parts to calculate

(a)

$$\int \cos^2(x)dx$$

(b)

$$\int x\cos(2x)\,dx$$

(c)

$$\int \sinh^{-1}(x)\,dx$$

(d)

$$\int \log_a(x)\,dx$$

(e)

$$\int \sin(\sqrt{x})\,dx$$

Problem 4 Suppose f and g are smooth functions with zeros at $x = a$ and $x = b$. Show that

$$\int_a^b \frac{df}{dx} g(x)dx = -\int_a^b f(x)\frac{dg}{dx}dx.$$

Problem 5 Integrate.

(a)

$$\int \left(5\sin^5(x)+3\sin^3(x)+\sin(x)\right)\,dx$$

(b)

$$\int \cos^2(x)\sin^2(x)\,dx$$

(c)

$$\int \csc(x)\,dx$$

(d)

$$\int x\sin(2x)\sin(7x)\,dx$$

Problem 6 Integrate.

(a)

$$\int \sqrt{x^2+4x+5}dx$$

(b)

$$\int 2^x\sqrt{2^{2x}+2^{x+2}+5}dx$$

Problem 7 Integrate, best answer best credit.

$$\int \frac{x^5dx}{\sqrt{64-4x^2}}$$

Problem 8 Integrate, best answer best credit.

(a)

$$\int x\sqrt{3-2x-x^2}\,dx$$

(b)

$$\int \frac{x^2}{\sqrt{1+x^2}}\,dx$$

Problem 9 Integrate these proper rational functions. Show all steps.

(a)

$$\int \frac{x\,dx}{x-2}$$

(b)

$$\int \frac{x\ dx}{x^2+2x+1}$$

(c)

$$\int \frac{(5-3x)\ dx}{x^2+4x+5}$$

(d)

$$\int \frac{(2x^2+3)dx}{x^4-2x^2+1}$$

Problem 10 Integrate these improper rational functions. Show all steps.

(a)

$$\int \frac{x^4}{x^2+4x+4}dx$$

(b)

$$\int \frac{x^3}{x^2+x-6}dx$$

Problem 11 Use Mathematica or some other CAS to find the partial fractions decomposition for the rational functions below:

(a)

$$\frac{32x^5 + 16x^4 + 19x^3 - 98x^2 - 107x - 15}{(x^2 - 2x - 15)(4x^2 + 4x + 5)^2}$$

(b)

$$\frac{1}{x^3 + 1}$$

Problem 12 Calculate bounds for $f(x), f''(x)$ and $f^{(4)}(x)$ on $0 \leq x \leq 2$ provided that $f(x) = e^{x^2}$. Notice that I know these functions are bounded since it is clear from the outset that these will be continuous functions and $[0, 2]$ is a closed interval. A well-known theorem of calculus states that a continuous function attains its extrema on a closed interval. Note that I don't ask you find the extrema, I just ask you find (and explain) why $f(x), f''(x)$ and $f^{(4)}(x)$ are bounded on $0 \leq x \leq 2$. This requires both calculation and thinking. Good hunting.

Problem 13 In this problem, use either Mathematica, an online calculator, or some other CAS and include 8 decimal places in your answers. You are to calculate $\int_0^2 e^{x^2}\,dx$ by the following numerical methods (put answer in box provided):

(a) left endpoint rule with $n = 10$, $\boxed{L_{10} = \qquad .}$

(b) right endpoint rule with $n = 10$, $\boxed{R_{10} = \qquad .}$

(c) trapezoid rule with $n = 10$, $\boxed{T_{10} = \qquad .}$

(d) midpoint rule with $n = 10$, $\boxed{M_{10} = \qquad .}$

(e) Simpson's rule with $n = 10$. $\boxed{S_{10} = \qquad .}$

Furthermore, I happen to know that $\int_0^2 e^{x^2}\,dx \approxeq 16.452627765507238$. Compare and contrast the outcome of the various approximation schemes. Do your observations agree with the error estimate theorems in Stewart? [use the bounds found in Problem 12 to properly apply the error theorems] Which is closest to the real answer (assume that I have given you the real answer)

Problem 14 Integrate. Indicate any interesting limits explicitly.

(a)

$$\int_2^3 \frac{dx}{\sqrt{x-2}}$$

(b)

$$\int_0^1 \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx$$

(c)

$$\int_0^2 \frac{dx}{(2x-1)^{2/3}}$$

Problem 15 Integrate. Indicate any interesting limits explicitly.

(a)

$$\int_0^\infty \frac{dx}{x^2+4}$$

(b)

$$\int_0^\infty \frac{x\,dx}{x^2+4}$$

(c)

$$\int_0^\infty \frac{x^2\,dx}{3^x}$$

Problem 16 Use the comparison theorem, tail theorem, linearity or the definition to determine if the integrals below converge or diverge. Mention any inequalities which justify your logic explicitly. Don't just say converge or diverge. Explain yourself briefly, but explicitly.

(a)

$$\int_1^\infty \frac{dx}{x^2+7}$$

(b)

$$\int_{10}^\infty \frac{dx}{x^2}$$

(c)

$$\int_1^\infty \frac{dx}{2x-|\sin(x)|}$$

(d)

$$\int_3^\infty \frac{dx}{x^2\ln(x)}$$

(e)

$$\int_{-\infty}^\infty \sin(x)\,dx$$