Please work the problems in the white space provided and clearly box your solutions. If there is not enough space please write the answer in the given space and indicate that the work is supplied on a separate sheet. Be sure to label attached sheets with the Problem number so the grader can efficiently judge your solution. Enjoy!

Problem 17 Suppose C is the set of all points $(x, y) \in \mathbb{R}^2$ which solve $xy - y = 1$. The goal of this problem is simply to examine how C can be described as a level-set, graph or parametrized curve:

(a) find $F : \mathbb{R}^2 \to \mathbb{R}$ such that $F^{-1}(\{0\})$ is C

(b) find $f : dom(f) \subseteq \mathbb{R} \to \mathbb{R}$ such that $graph(f) = C$

(c) find $x : J_1 \subseteq \mathbb{R} \to \mathbb{R}$ and $y : J_1 \subseteq \mathbb{R} \to \mathbb{R}$ such that $\{(x(t), y(t)) \mid t \in J_1\} \subset C$ (choose parametric equations which cover the right-branch of C)

(d) find $x : J_2 \subseteq \mathbb{R} \to \mathbb{R}$ and $y : J_2 \subseteq \mathbb{R} \to \mathbb{R}$ such that $\{(x(t), y(t)) \mid t \in J_2\} \subset C$ (choose parametric equations which cover the left-branch of C)

(e) use part b to write $\vec{r} : J_3 \to \mathbb{R}^2$ which covers all of C.

Problem 18 Suppose $x = t^4 - 2t^3 + t^2$ and $y = t^2 - t$ for $t \in [-2, 4]$ is a parametrization of the curve C.

(a) make a table of values for $t = -2, -1, 0, 1, 2, 3, 4$ and plot the curve by plotting these points connecting the dots as smoothly as possible.

(b) find the Cartesian form of the curve C.

(c) find $\vec{r} : [-2, 4] \to \mathbb{R}^2$ such that $\vec{r}([-2, 4]) = C$.

(d) calculate $\frac{d\vec{r}}{dt}$ and find any critical points for the path.

Problem 19 Creating parametric equations for given level curves is generally an art. However, the questions that follow are either in my notes or are really close to something I do show: in each case find parametric equations and state the domain for the parameter which will reproduce the given curve including any constraints, you are free to use either vector or scalar notation (you can just state the answers on this page, I don't expect derivations here)

(a) find the parametrization of the linear path from $(0, 1)$ to $(4, -3)$.

(b) find the parametrization of the linear path from $(4, -3)$ to $(0, 1)$.

(c) find the parametric equations for $x^2 - 2x + y^2 = 0$ for $y > \sqrt{3}/2$ where the curve is oriented in the CCW-direction.

(d) find the parametric equations for $x^2 - 2x + y^2 = 0$ for $y > \sqrt{3}/2$ where the curve is oriented in the CW-direction.

(e) find parametric equations for the upper branch of $x^2 - 2(y - 2)^2 = 1$.

(f) find the parametric equations which locally cover for $y^2x^2 = 1$ in a neighborhood of $(-1, -1)$. (Try using x as a parameter)

Problem 20 Vectors have both magnitude and a direction. This direction is conveniently described by the so-called standard angle. For this problem, make your standard angles fall in the range $\theta \in [0, 2\pi)$. You'll need to do some thinking about trigonometry here, the calculator might let you down. Draw pictures to be safe.

(a) $\vec{A} = \langle 1, 3 \rangle$

(b) $\vec{B} = \langle 2, -3 \rangle$

(c) $\vec{C} = \langle -3, 0 \rangle$

(d) $\vec{D} = \langle -2, -2 \rangle$

(e) plot all four vectors in a single plot.

Problem 21 Vectors correspond to points in a very natural manner. For each of the points below find the distance from the origin to the point and the standard **polar** angle of the given point:

(a) $A = (1, 3)$

(b) $B = (2, -3)$

(c) $C = (-3, 0)$

(d) $D = (-2, -2)$

(e) plot all four points in a single plot.

Problem 22 Suppose $\vec{r}(t) = \langle\ t\sin(t),\ t\sin(t)\ \rangle$ denotes the position of a particle at time t.

(a) find the velocity at time t

(b) find the acceleration at time t

(c) find the displacement vector from $t = 0$ to $t = 2\pi$

(d) find the distance travelled from $t = 0$ to $t = 2\pi$

Problem 23 Suppose $\vec{a}(t) = \langle 0, -g \rangle$ is the acceleration of a particle by gravity near the surface of the earth. Futhermore, suppose the particle begins at the origin with velocity $\vec{v}(0) = \langle v_{ox}, v_{oy} \rangle$. Show that the Cartesian form of the path is a parabola. Your answer will involve the initial velocity components and g.

Problem 24 Suppose $\vec{a}(t) = \langle 2, 5 \rangle$ is the acceleration and $\vec{v}(1) = \langle 3, -2 \rangle$ and $\vec{r}(0) = \langle 1, 1 \rangle$. Find:

(a) $\vec{v}(t)$ (the velocity vector at time t)

(b) $\vec{r}(t)$ (the position vector at time t)

(c) $s(t)$ where we take $t = 0$ as our base point. This is the arclength of the path from $t = 0$ to time $t > 0$, in other words $s(t)$ is the distance travelled during $[0, t]$.

Problem 25 Suppose $\vec{r}(t) = \left\langle\, 2\sin(3t),\ -3\sin(2t)\,\right\rangle$. Find all:

$(a.)$ horizontal tangents, $(b.)$ vertical tangents, $(c.)$ critical points

Then (d.) plot the curve (use CAS or draw neatly by hand).

Problem 26 On parametric calculus:

(a) Problem 40 from Section 11.2 of Stewart.

(b) Problem 65 from Section 11.2 of Stewart.

(c) Problem 69 from Section 11.2 of Stewart.

(d) Problem 74 from Section 11.2 of Stewart.

Problem 27 On arclength:

(a) $y = \ln(\sec(x))$ for $0 \le x \le \pi/4$.

(b) the path with parametrization $\vec{r}(\lambda) = \left\langle \lambda^2, \lambda^2 \right\rangle$ for $\lambda \in [0, 2]$.

(c) the path with parametrization $x = 2^t \cos(t)$ and $y = 2^t \sin(t)$ for $t \in [0, \pi/2]$

(d) Problem 38 from Section 9.1 of Stewart.

Problem 28 On surface area:

(a) Problem 8 from Section 9.2 of Stewart.

(b) Problem 16 from Section 9.2 of Stewart.

(c) Problem 28 from Section 9.2 of Stewart.

Problem 29 On physics:

(a) Problem 10 from Section 9.3 of Stewart.

(b) Problem 30 from Section 9.3 of Stewart.

(c) Problem 23 from Section 6.4 of Stewart.

Problem 30 [worth 3pts, 2 of which are bonus!] On polar geometry:

(a) Problem 10 from Section 11.3 of Stewart.

(b) Problem 20 from Section 11.3 of Stewart.

(c) Problem 24 from Section 11.3 of Stewart.

(d) Problem 30 from Section 11.3 of Stewart.

(e) Problem 42 from Section 11.3 of Stewart.

(f) Problem 60 from Section 11.3 of Stewart.

(g) Problem 66 from Section 11.3 of Stewart.

(h) Problem 70 from Section 11.3 of Stewart.

(i) Problem 79 from Section 11.3 of Stewart.

Problem 31 On polar coordinate-based calculus:

(a) Problem 12 from Section 11.4 of Stewart.

(b) Problem 18 from Section 11.4 of Stewart.

(c) Problem 26 from Section 11.4 of Stewart.

Problem 32 On polar coordinate-based calculus:

(a) Problem 30 from Section 11.4 of Stewart.

(b) Problem 38 from Section 11.4 of Stewart.

(c) Problem 46 from Section 11.4 of Stewart.