EXAMPLES OF P-SERIES

$$\boxed{\text{Th}^{\underline{m}}/ \quad \sum_{n=1}^{\infty} \frac{1}{n^p} \text{ converges iff } p > 1.}$$

1.) $S = \sum_{n=1}^{\infty} \frac{1}{n}$ is $p = 1 \not> 1$ $\therefore$ S diverges by $p=1$ SERIES TEST.

(aka HARMONIC SERIES)

2.) $S = \sum_{n=3}^{\infty} \frac{1}{n^2}$ is almost the $p=2$ SERIES

notice $S + 1 + \frac{1}{4} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ ← $p=2$ SERIES

$\therefore$ CONVERGES BY $p=2$ SERIES TEST.

It follows S converges.

In fact, as $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ (by MATH MAGIC FOR NOW)

we find $\boxed{S = \frac{\pi^2}{6} - \frac{5}{4}}$

intuition: conv/div cares about tail of series

$$\boxed{\text{Th}^{\underline{m}}/ \quad \sum_{n=n_0}^{\infty} \frac{1}{n^p} \text{ converges iff } p > 1 \text{ where } n_0 \in \mathbb{N} \text{ is some fixed \#.}}$$

(the $\text{Th}^{\underline{m}}$ above generalizes the argument of 2. where $n_0 = 3$ and $p = 2$).

3.) $\sum_{n=13}^{\infty} \frac{1}{\sqrt{n}}$ is almost a $p = 1/2 < 1$ SERIES

thus it diverges by $p = 1/2$ series test

4.) $$\sum_{n=1}^{\infty}\left(\frac{1}{n^3}+\frac{1}{n\sqrt{n}}\right) \overset{*}{=} \underbrace{\sum_{n=1}^{\infty}\frac{1}{n^3}}_{\substack{P=3 \\ \text{CONVERGES} \\ \text{BY P-SERIES} \\ \text{TEST}}} + \underbrace{\sum_{n=1}^{\infty}\frac{1}{n^{1.5}}}_{\substack{P=1.5 \\ \text{CONVERGES} \\ \text{BY P-SERIES} \\ \text{TEST}}}$$

$*$: this step allowed because the RHS series converge.

$$\boxed{\text{Th}^{m}/ \text{ If } a_n, b_n \geq 0 \text{ and } \sum' a_n \text{ or } \sum' b_n \text{ diverge then } \sum' (a_n + b_n) \text{ likewise diverge.}}$$

This notation is used when starting point of series is immaterial to the argument.

5.) $$S' = \sum_{n=3}^{\infty}\left(\frac{n}{\sqrt[3]{n}}+\frac{1}{n^3}\right) \text{ diverges}$$

Since $\frac{n}{\sqrt[3]{n}}, \frac{1}{n^3} \geq 0$ and

$$\sum_{n=3}^{\infty}\frac{n}{\sqrt[3]{n}} = \sum_{n=3}^{\infty}\frac{1}{n^{-2/3}} \leftarrow \underbrace{P = \frac{-2}{3} < 1}_{\text{divergent P-series.}}$$

<u>Alternatively</u>: S' diverges as $\frac{n}{\sqrt[3]{n}} + \frac{1}{n^3} \longrightarrow \infty \neq 0$ as $n \longrightarrow \infty$ thus S' <u>fails</u> n^{th} term test.

6.) $$S' = \sum_{n=1}^{\infty}\frac{3}{n^3} = 3\underbrace{\left(\sum_{n=1}^{\infty}\frac{1}{n^3}\right)}_{P=3,\ \text{CONVERGES BY P-SERIES TEST.}} \therefore S' \text{ converges.}$$