Printed Name: ______________________.

PHYSICS 331	MISSION 5: MAGNETOSTATICS (38+2)PTS)

Problems mentioned refer to David Griffith's *Introduction to Electrodynamics*, 5th edition. Skip two problems. Each problem worth 4pts.

Problem 42 Problem 5.9 (magnetic field due some standard current configurations)

Problem 43 Problem 5.11 (finite solenoid)

Problem 44 Problem 5.13 (magnetic vs. electric force on two wires)

Problem 45 Problem 5.14 (Ampere's Law straight wire)

Problem 46 Problem 5.15 (Ampere's Law infinite slab current)

Problem 47 Problem 5.20 (independence of bounding surface in steady current case)

Problem 48 Problem 5.25 (magnetic vector potential, straight line segment)

Problem 49 Problem 5.27 (magnetic vector potential for uniform magnetic field)

Problem 50 Problem 5.29 (magnetic vector potential, plane surface current)

Problem 51 Problem 5.31 (concerning the weirdness of a magnetic scalar potential)

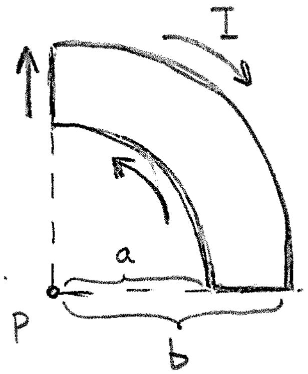
Problem 52 Problem 5.37 (magnetic dipole moment of current loop and its connection to pure dipole in far-field limit)

Problem 53 Problem 5.60 (magnetic dipole in otherwise uniform magnetic field)

Solution to Mission 5

$\boxed{P42}$ Problem 5.9

Find the magnetic field at the point P for the steady currents pictured below:



$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$\hat{s} \to \hat{\phi} \to \hat{z} \to \hat{s}$

$$d\vec{l}_a = a\,d\phi\,\hat{\phi}$$

$$\hat{\mathscr{r}}_a = -\hat{s}\ ,\ \mathscr{r} = a$$

$$d\vec{l}_a \times \hat{\mathscr{r}} = -a\,d\phi\,\hat{\phi}\times\hat{s} = a\,d\phi\,\hat{z}$$

$$d\vec{l}_b = b\,d\phi\,\hat{\phi}$$

$$\hat{\mathscr{r}}_b = -\hat{s}$$

$$\mathscr{r} = b$$

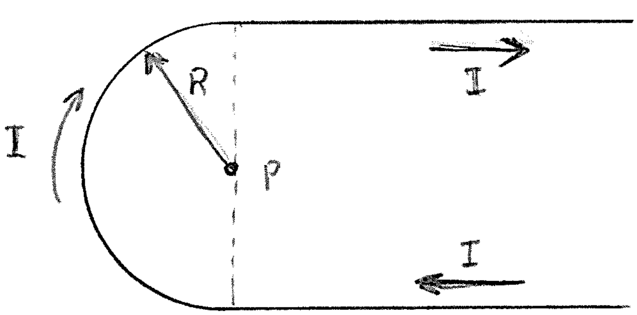
$$d\vec{l}_b \times \hat{\mathscr{r}} = b\,d\phi\,\hat{z}$$

Remark: the radial current segments produce no field at P since $\hat{\mathscr{r}} \parallel \pm d\vec{l}$

$$\vec{B}(0) = \frac{\mu_0 I}{4\pi}\int_0^{\pi/2}\frac{a\,d\phi\,\hat{z}}{a^2} + \frac{\mu_0 I}{4\pi}\int_{\pi/2}^{0}\frac{b\,d\phi\,\hat{z}}{b^2}$$

$$\vec{B}(0) = \frac{\mu_0 I}{4\pi}\left(\frac{1}{a}\left(\frac{\pi}{2}\right) - \frac{1}{b}\left(\frac{\pi}{2}\right)\right)\hat{z}$$

$$\boxed{\vec{B}(P) = \frac{\mu_0 I}{8}\left(\frac{1}{a} - \frac{1}{b}\right)\hat{z}} \quad \left(\text{out of page}\right)$$



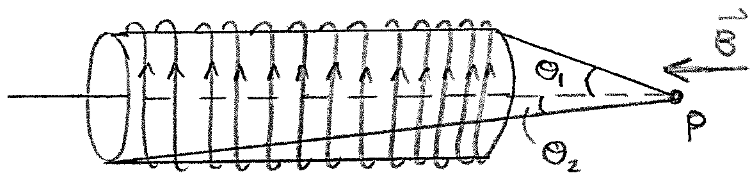
$$d\vec{l} = -dl\,\hat{\phi}\ ,\ \hat{\mathscr{r}} = -\hat{s}\ ,\ \mathscr{r} = R$$

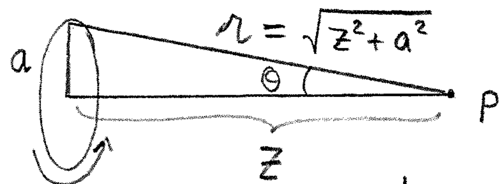
$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{dl\,\hat{\phi}\times\hat{s}}{R^2} = \frac{\mu_0 I}{4\pi}\left(\frac{\pi R(-\hat{z})}{R^2}\right)$$

$$\boxed{\vec{B}(P) = -\frac{\mu_0 I\hat{z}}{4R}} \quad (\text{into page})$$

P43 Problem 5.11

Find magnetic field at P on axis of tightly wound solenoid of n turns per meter carrying current I with radius a. Find field for infinite solenoid ($\theta_2 \to 0$, $\theta_1 \to \pi$)





$$\sin\theta = \frac{a}{\sqrt{a^2+z^2}} \qquad \cos\theta = \frac{z}{\sqrt{a^2+z^2}}$$

$dI = nI\,dz \longleftarrow n\,dz = \#$ of loops within dz at z

$$dB = \frac{\mu_0\, dI}{2}\frac{a^2}{(a^2+z^2)^{3/2}} = \frac{\mu_0 n I}{2}\left(\frac{a^2\,dz}{(a^2+z^2)^{3/2}}\right)$$

Now, I need to express dz in terms of $d\theta$

$$\cos\theta\, d\theta = -\frac{a}{2}(a^2+z^2)^{-3/2}(2z)\,dz = \frac{-az\,dz}{(a^2+z^2)^{3/2}}$$

Thus, $dz/(z^2+a^2)^{3/2} = \frac{-1}{az}\cos\theta\, d\theta$ and so,

$$dB = \frac{-\mu_0 n I}{2}\frac{a^2}{az}\cos\theta\, d\theta$$

Notice $\tan\theta = a/z$ hence,

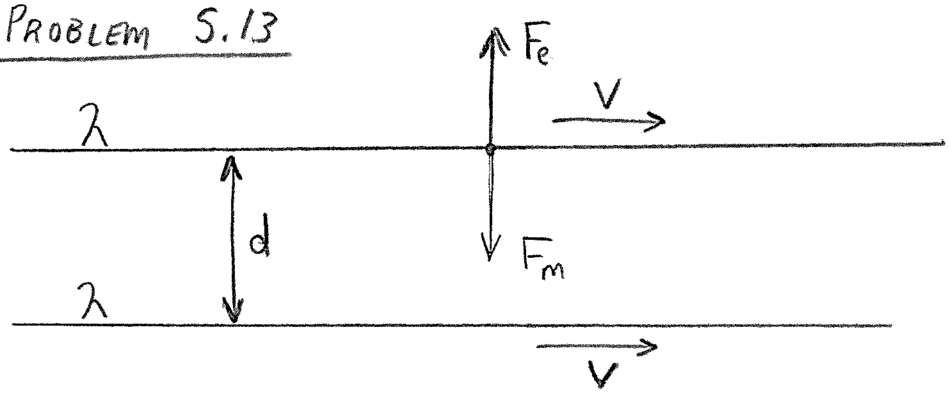
$$dB = -\left(\frac{\mu_0 n I}{2}\right)\tan\theta\cos\theta\, d\theta = \frac{-\mu_0 I n}{2}\sin\theta\, d\theta$$

$$\therefore \quad B = \frac{-\mu_0 n I}{2}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$$

$$\therefore \quad \boxed{B = \frac{\mu_0 I n}{2}\left(\cos\theta_2 - \cos\theta_1\right)}$$

Infinitely long solenoid, $B = \frac{\mu_0 I n}{2}(\cos 0 - \cos\pi) = \boxed{\mu_0 n I}$

P44) PROBLEM 5.13



Consider a particular length l

$$B = \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 \lambda v}{2\pi d}$$

$$F_m = \|I\vec{l} \times \vec{B}\| = \frac{\mu_0 l(\lambda v)(\lambda v)}{2\pi d}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 d}$$

$$F_E = QE = \frac{\lambda l \lambda}{2\pi\epsilon_0 d}$$

$$F_m = F_e$$

$$\frac{\mu_0 l \lambda^2 v^2}{2\pi d} = \frac{\lambda^2 l}{2\pi\epsilon_0 d}$$

$$v^2 = \frac{1}{\mu_0\epsilon_0}$$

$$\boxed{v = \frac{1}{\sqrt{\mu_0\epsilon_0}}}$$ (speed of light)

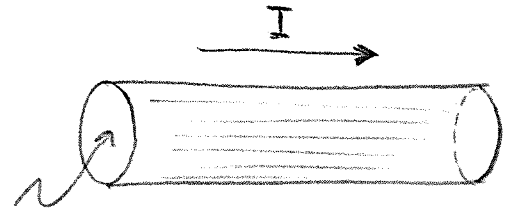
P45 PROBLEM 5.14

A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field both inside and outside of the wire if

(a.) the current is uniformly distributed over surface of wire

(b.) the current is distributed s.t. J is proportional to s

(a.)



$\vec{J} = 0$ for $0 \leq s < a$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$2\pi s B = \mu_0 I_{enc}$$

$$\boxed{\vec{B} = \begin{cases} 0 & \text{for } 0 \leq s < a \\ \dfrac{\mu_0 I}{2\pi s}\hat{\phi} & \text{for } s > a \end{cases}}$$

(b.) $I = \int \alpha s \, dA = \int_0^{2\pi}\int_0^a \alpha s^2 \, ds \, d\phi = \dfrac{2\pi \alpha a^3}{3} = I$

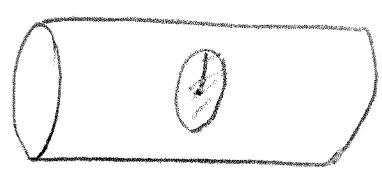
Thus $J = \alpha s = \left(\dfrac{3I}{2\pi a^3}\right) s$

$$I_{enc}(s \leq s_0) = \int_0^{2\pi}\int_0^{s_0} \frac{3Is}{2\pi a^3} s \, ds \, d\phi$$

$$= \left(\frac{3I}{2\pi a^3}\right)\left(\int_0^{2\pi} d\phi\right)\left(\int_0^{s_0} s^2 ds\right)$$

$$= \left(\frac{3I}{a^3}\right)\left(\frac{s_0^3}{3}\right)$$

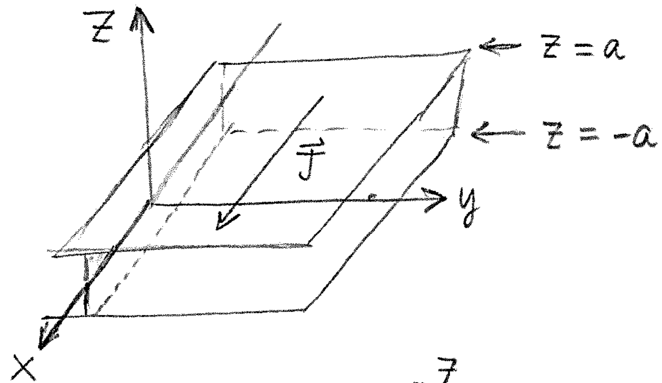
$$= I s_0^3 / a^3$$



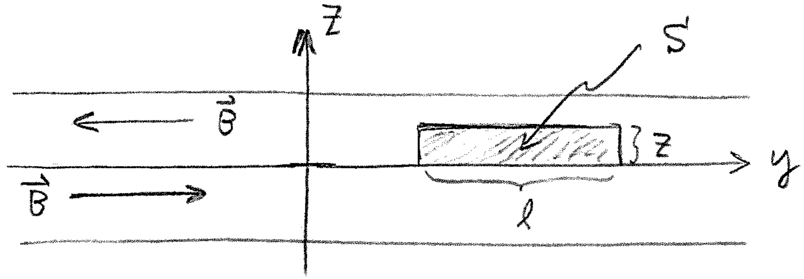
$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \implies (2\pi s) B = \mu_0 I s^3/a^3 \quad \therefore \; B = \dfrac{\mu_0 I s^2}{2\pi a^3}$

$$\implies \boxed{\vec{B} = \begin{cases} \dfrac{\mu_0 I s^2}{2\pi a^3} & \text{for } 0 \leq s \leq a \\ \dfrac{\mu_0 I}{2\pi s} & \text{for } s \geq a \end{cases}}$$

P46 PROBLEM 5.15



$$\vec{J} = J\hat{x}$$

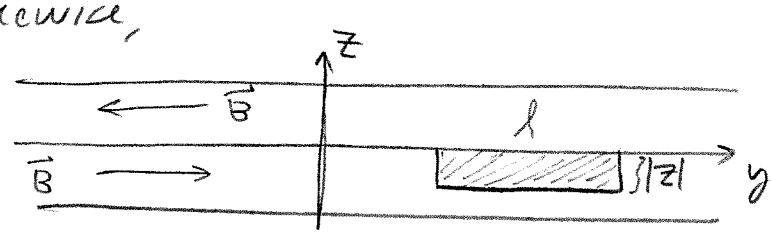


<u>Remark</u>: $\vec{B} = 0$ on the xy-plane.

$$\mu_0 \int_S \vec{J} \cdot d\vec{a} = \oint_{\partial S} \vec{B} \cdot d\vec{\ell} \Rightarrow \mu_0 J \ell z = B\ell$$ (only top edge contributes)

$$\boxed{B = -\mu_0 J z \hat{y} \text{ for } 0 \leq z \leq a}$$

Likewise,



$$\mu_0 \int_S \vec{J} \cdot d\vec{a} = \oint_{\partial S} \vec{B} \cdot d\vec{\ell} \Rightarrow \mu_0 J \ell z = B\ell$$ (only base edge gives nonzero contribution)

$$\boxed{\vec{B} = \mu_0 J |z| \hat{y}}$$

for $-a \leq z \leq 0$.

Notice we can extend the amperian loops beyond the slab and $I_{enc} = J\ell a$ in both cases,

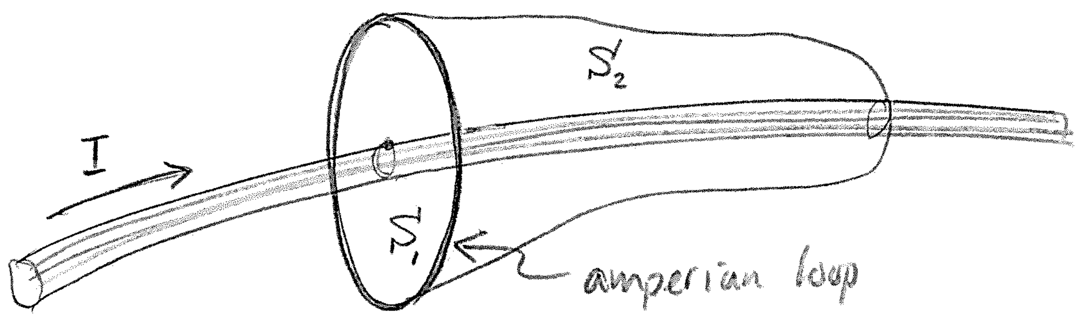
$$\boxed{\vec{B} = \begin{cases} -\mu_0 J z \hat{y} & : -a \leq z \leq a \\ -\mu_0 J a \hat{y} & : z \geq a \\ \mu_0 J a \hat{y} & : z \leq -a \end{cases}}$$

P47 PROBLEM 5.20

In calculating the current enclosed by an Amperian loop, one must, in general, evaluate an integral of the form

$$I_{enc} = \int_S \vec{J} \cdot d\vec{a}$$

Trouble is, there are infinitely many surfaces that share the same boundary line. Which one are we supposed to use?



Notice $S_1 \cup (-S_2) = \partial \mathcal{V}$ and

$$I_{enc}(S_1) = \int_{S_1} \vec{J} \cdot d\vec{a}$$

$$I_{enc}(S_2) = \int_{S_2} \vec{J} \cdot d\vec{a}$$

$$\int_{\mathcal{V}} (\nabla \cdot \vec{J})\, d\tau = \int_{\partial \mathcal{V}} \vec{J} \cdot d\vec{a} = \int_{S_1} \vec{J} \cdot d\vec{a} - \int_{S_2} \vec{J} \cdot d\vec{a}$$

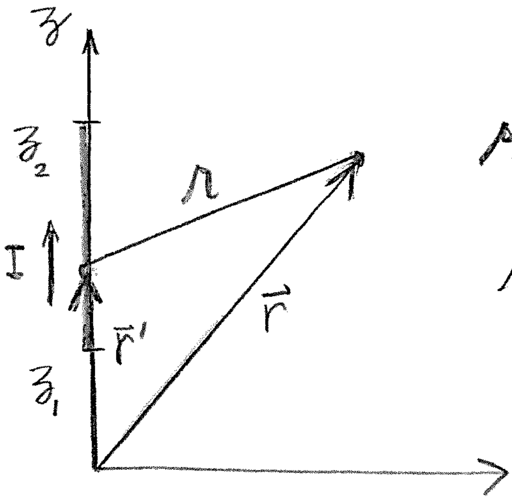
Steady current $\therefore$ $\nabla \cdot \vec{J} = 0$ and so,

$$\underline{I_{enc}(S_1) = I_{enc}(S_2)}$$

(it matters not which bounding surface for the amp. loop, all give same current)

P48 PROBLEM 5.25

Find the magnetic vector potential of a finite segment of straight wire carrying current I. (Put the wire on the z-axis, from z_1 to z_2 and use Eq 5.66) Check your answer is consistent with 5.37



$$\mathscr{r} = \sqrt{x^2+y^2+(z-z')^2}$$

$$\mathscr{r} = \sqrt{s^2+(z-z')^2}$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\int \frac{1}{\mathscr{r}}\,d\vec{l}'$$

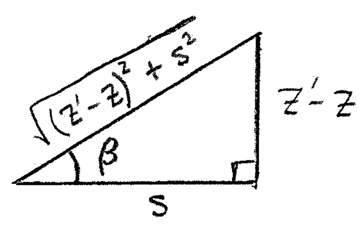
$$= \frac{\mu_0 I \hat{z}}{4\pi}\int_{z_1}^{z_2} \frac{dz'}{\sqrt{(z'-z)^2+s^2}}$$

$z'-z = s\tan\beta$

$(z'-z)^2+s^2 = s^2(\tan^2\beta+1) = s^2\sec^2\beta$

$dz' = s\sec^2\beta\,d\beta$

$$= \frac{\mu_0 I \hat{z}}{4\pi}\int_{\beta_1}^{\beta_2} \frac{s\sec^2\beta\,d\beta}{s\sec\beta}$$



$$= \frac{\mu_0 I \hat{z}}{4\pi}\int_{\beta_1}^{\beta_2} \sec\beta\,d\beta$$

$$= \frac{\mu_0 I \hat{z}}{4\pi}\left(\ln|\sec\beta_2+\tan\beta_2| - \ln|\sec\beta_1+\tan\beta_1|\right)$$

$$= \frac{\mu_0 I \hat{z}}{4\pi}\left(\ln\left|\frac{\sqrt{(z_2-z)^2+s^2}}{s}+\frac{z_2-z}{s}\right| - \ln\left|\frac{\sqrt{(z_1-z)^2+s^2}+z_1-z}{s}\right|\right)$$

$$= \frac{\mu_0 I \hat{z}}{4\pi}\left(\ln\left|\sqrt{(z-z_2)^2+s^2}+z_2-z\right| - \ln\left|\sqrt{(z-z_1)^2+s^2}+z_1-z\right|\right)$$

P48 continued

$$\vec{B} = \nabla \times \vec{A} = \cancel{\frac{1}{s}\frac{\partial A_z}{\partial \phi}\hat{s}}^{\,0} - \frac{\partial A_z}{\partial s}\hat{\phi} \qquad \left(\text{other terms zero since } A_\phi = A_s = 0\right)$$

$$\vec{B} = -\frac{\partial}{\partial s}\left[\ln\left|\sqrt{(z-z_2)^2+s^2}+z_2-z\right| - \ln\left|\sqrt{(z-z_1)^2+s^2}+z_1-z\right|\right]\frac{\mu_0 I}{4\pi}\hat{\phi}$$

$$= \left[\left(\frac{1}{\sqrt{(z-z_1)^2+s^2}+z_1-z}\right)\left(\frac{2s}{2\sqrt{(z-z_1)^2+s^2}}\right)\left(\frac{\mu_0 I}{4\pi}\right) + \right.$$

$$\left. - \left(\frac{1}{\sqrt{(z-z_2)^2+s^2}+z_2-z_1}\right)\left(\frac{2s}{2\sqrt{(z-z_2)^2+s^2}}\right)\left(\frac{\mu_0 I}{4\pi}\right)\right]\hat{\phi}$$

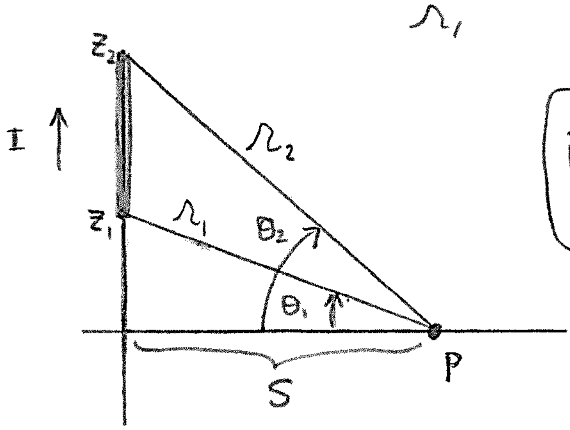
Now, to compare with Eq^n 5.37 we need to set $z = 0$ in our current notation

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left[\left(\frac{1}{\sqrt{z_1^2+s^2}+z_1}\right)\left(\frac{s}{\sqrt{z_1^2+s^2}}\right) - \left(\frac{1}{\sqrt{z_2^2+s^2}+z_2}\right)\left(\frac{s}{\sqrt{z_2^2+s^2}}\right)\right]\hat{\phi}$$

$$= \frac{\mu_0 I}{4\pi}\left[\frac{z_1-\sqrt{z_1^2+s^2}}{-(z_1^2+s^2)+z_1^2}\cdot\frac{s}{\sqrt{z_1^2+s^2}} - \frac{z_2-\sqrt{z_2^2+s^2}}{-(z_2^2+s^2)+z_2^2}\cdot\frac{s}{\sqrt{z_2^2+s^2}}\right]\hat{\phi}$$

$$= \frac{\mu_0 I}{4\pi}\left[\frac{z_1-\sqrt{z_1^2+s^2}}{-s^2}\cdot\frac{s}{\sqrt{z_1^2+s^2}} - \frac{z_2-\sqrt{z_2^2+s^2}}{-s^2}\cdot\frac{s}{\sqrt{z_2^2+s^2}}\right]\hat{\phi}$$

$$= \frac{\mu_0 I}{4\pi s}\left[\frac{-z_1}{\underbrace{\sqrt{z_1^2+s^2}}_{\mathscr{r}_1}} + 1 + \frac{z_2}{\underbrace{\sqrt{z_2^2+s^2}}_{\mathscr{r}_2}} - 1\right]\hat{\phi}$$



$$\boxed{\vec{B}(P) = \frac{\mu_0 I}{4\pi s}\left[\sin\theta_2 - \sin\theta_1\right]\hat{\phi}}$$

P49 PROBLEM 5.27

Suppose $\vec{B}$ is uniform, show that $\vec{A}(\vec{r}) = \frac{-1}{2}\vec{r} \times \vec{B}$ works. That is, check that $\nabla \cdot \vec{A} = 0$ and $\nabla \times \vec{A} = \vec{B}$. Is this choice of $\vec{A}$ unique?

There may be more clever solutions, but

$$\vec{A}(\vec{r}) = \frac{-1}{2}\langle x, y, z\rangle \times \langle B_1, B_2, B_3\rangle$$

$$= \frac{-1}{2}\langle yB_3 - zB_2,\ zB_1 - xB_3,\ xB_2 - yB_1\rangle$$

Thus clearly, $\underline{\nabla \cdot \vec{A} = 0}$. Moreover,

$$\nabla \times \vec{A} = \frac{-1}{2}\Big[\Big(\Big[\frac{\partial}{\partial y}[xB_2 - yB_1] - \frac{\partial}{\partial z}[zB_1 - xB_3]\Big]\Big)\hat{x}$$

$$+\Big(\Big[\frac{\partial}{\partial z}[yB_3 - zB_2] - \frac{\partial}{\partial x}[xB_2 - yB_1]\Big]\Big)\hat{y}$$

$$+\Big(\Big[\frac{\partial}{\partial x}[zB_1 - xB_3] - \frac{\partial}{\partial y}[yB_3 - zB_2]\Big]\Big)\hat{z}\Big]$$

$$= \frac{-1}{2}\left(-2B_1\hat{x} - 2B_2\hat{y} - 2B_3\hat{z}\right)$$

$$= B_1\hat{x} + B_2\hat{y} + B_3\hat{z}$$

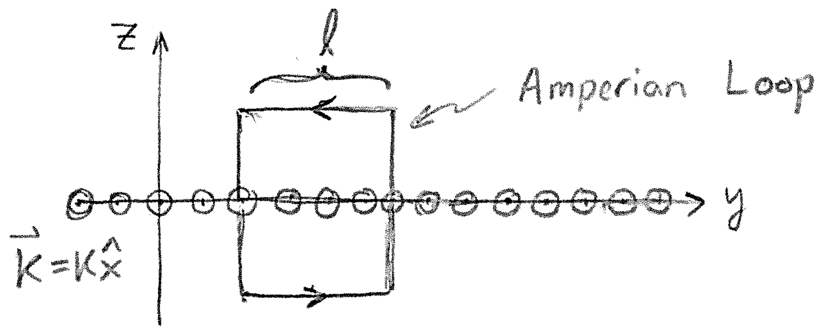
$$= \vec{B}.$$

No, the choice is <u>not</u> unique if we pick λ for which $\nabla \cdot (\nabla\lambda) = 0$ then $\nabla \cdot (\vec{A} + \nabla\lambda) = 0$ and $\nabla \times (\vec{A} + \nabla\lambda) = \nabla \times \vec{A} + \nabla \times \nabla\lambda = \vec{B} + \vec{0} = \vec{B}$.

There are <u>many</u> solutions to $\nabla^2\lambda = 0$.

P50 Problem 5.29

Find vector potential above and below plane surface current of the form $\vec{K} = K\hat{x}$ flowing over xy-plane



RHR & symmetry gives $\vec{B}$ pointing left for $z>0$ and right for $z<0$.

$$\oint_{\partial S} \vec{B}\cdot d\vec{l} = \mu_0 \int_S \vec{J}\cdot d\vec{a} = \mu_0 K l = Bl + Bl$$

$$B = \frac{\mu_0 K}{2}$$

Therefore,

$$\vec{B} = \begin{cases} \frac{\mu_0 K}{2}\hat{y} & \text{for } z<0 \\ -\frac{\mu_0 K}{2}\hat{y} & \text{for } z>0 \end{cases}$$

I can use the previous problem, $\vec{A} = \frac{-1}{2}\vec{r}\times\vec{B}$

$$\vec{A} = \frac{-1}{2}(x\hat{x} + y\hat{y} + z\hat{z}) \times \left(\frac{\pm\mu_0 K}{2}\hat{y}\right)$$

$$= \mp\frac{\mu_0 K}{4}(x\hat{z} - z\hat{x})$$

$$\vec{A}(x,y,z) = \begin{cases} -\frac{\mu_0 K}{4}\langle -z, 0, x\rangle & : z<0 \\ \frac{\mu_0 K}{4}\langle -z, 0, x\rangle & : z>0 \end{cases}$$

However, we could also use (basically drop the x-terms and double-up the z-terms)

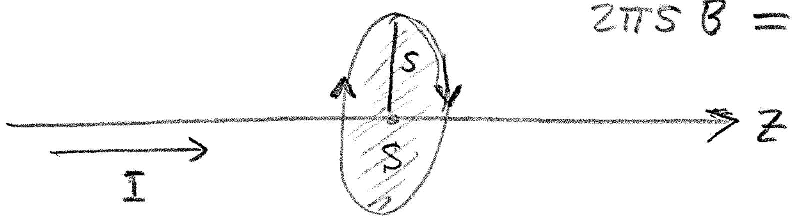
$$\boxed{\vec{A}(x,y,z) = \frac{-\mu_0 K}{2}|z|\hat{x}}$$

P51 Problem 5.31

Find the scalar potential of infinite straight wire
To avoid multivalued potential we must restrict our attention to simply connected region which does not encircle the current

$$\oint_S \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$2\pi s B = \mu_0 I$$



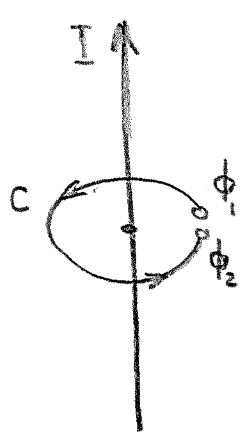
$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi} = -\nabla U = -\frac{\partial U}{\partial s}\hat{s} - \frac{1}{s}\frac{\partial U}{\partial \phi}\hat{\phi} - \frac{\partial U}{\partial z}\hat{z}$$

$$-\frac{1}{s}\frac{\partial U}{\partial \phi} = \frac{\mu_0 I}{2\pi s} \quad \text{and} \quad \frac{\partial U}{\partial s} = 0 \ \& \ \frac{\partial U}{\partial z} = 0$$

$$\frac{dU}{d\phi} = \frac{-\mu_0 I}{2\pi}$$

$$\boxed{U(\vec{r}) = \frac{-\mu_0 I \phi}{2\pi}}$$



$$\int_{C_1} \vec{B}\cdot d\vec{l} = -\int_{C_1} \nabla U \cdot d\vec{l}$$

$$= -U(\phi_2) + U(\phi_1)$$

$$= \frac{\mu_0 I}{2\pi}(\phi_2 - \phi_1) \longrightarrow \mu_0 I$$

in the limit $\phi_2 \longrightarrow \phi_1 + 2\pi$.

P52 Problem 5.37

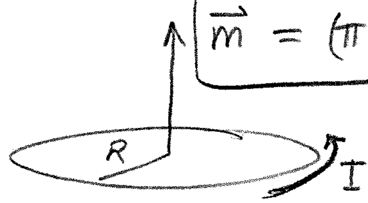
A circular loop of wire, with radius R, lies in the xy-plane (centered at the origin) and carries a current I running CCW as viewed from the positive z-axis.

(a.) What is its magnetic dipole moment?

(b.) What is the (approximate) magnetic fields at points far from the origin?

(c.) Show that, for points on the z-axis, your answer is consistent with the exact field (Ex. 5.6) when $z \gg R$.

(a.) $\boxed{\vec{m} = (\pi R^2 I)\hat{z}}$



(b.) $$\vec{B}_{dip}(\vec{r}) = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right) = \boxed{\frac{\mu_0 R^2 I}{4r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)}$$

(c.) Points on z-axis have $\theta = 0$ thus $\underbrace{\vec{B}_{dip} = \frac{\mu_0 R^2 I}{2z^3}\hat{z}}_{(r = z)}$

Example 5.6: field points in $\hat{z}$-direction and,

$$B = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$$

$z \gg R$

$$\cong \frac{\mu_0 I}{2}\frac{R^2}{(z^2)^{3/2}}$$

$$= \frac{\mu_0 I R^2}{2z^3} \checkmark$$

PS3 Problem 5.60

A magnetic dipole $\vec{m} = -m_0 \hat{z}$ is at origin in otherwise uniform magnetic field $\vec{B} = B_0 \hat{z}$. Show $\exists$ a spherical surface centered at origin through which no B-field lines pass. Find the radius of this sphere, and sketch the field lines inside & out.

$$\underbrace{\vec{B} = B_0 \hat{z} + \vec{B}_{dip}}_{\text{satisfies the given data hence must be the correct } \vec{B} \text{ field}}$$

(far field the $B_0 \hat{z}$ term dominates since $\| \vec{B}_{dip} \| \sim \frac{1}{r^3}$)

$$\vec{B} = B_0 \hat{z} - \frac{\mu_0 m_0}{4\pi r^3}\left(2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta}\right)$$

Thus calculate

$$\vec{B} \cdot \hat{r} = B_0 \hat{z} \cdot \hat{r} - \frac{\mu_0 m_0}{4\pi r^3}\left(2\cos\theta \cancelto{1}{\hat{r} \cdot \hat{r}} + \sin\theta \cancelto{0}{\hat{\theta} \cdot \hat{r}}\right)$$

Since $\hat{r} = \langle \cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta \rangle$ we see $\hat{z} \cdot \hat{r} = \cos\theta$

$$\vec{B} \cdot \hat{r} = B_0 \cos\theta - \frac{\mu_0 m_0 \cos\theta}{2\pi r^3}$$

$$\vec{B} \cdot \hat{r} = \left(B_0 - \frac{\mu_0 m_0}{2\pi r^3}\right) \cos\theta$$

Let R be such that $B_0 = \frac{\mu_0 m_0}{2\pi R^3}$ then B-field lines have no radial comp. at $r = R$ hence no field lines pass that radius.

$$\boxed{R = \sqrt[3]{\frac{\mu_0 m_0}{2\pi B_0}}}$$