Printed Name: ______________________________.

| PHYSICS 331 | TEST 2: MAGNETOSTATICS & ELECTRODYNAMICS (150PTS+10PTS) |
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*Please work the problems in the white space provided and clearly box your solutions. You are allowed a page of notes, front and back is fine. Enjoy!*

**Problem 1** (20pts) Suppose we define $\vec{A}$ for a given steady current density $\vec{J}$. In this problem we wish to justify the choice of Coulomb Gauge $\nabla \bullet \vec{A} = 0$. You are given the existence of a vector field $\vec{A}_o$ for which $\vec{B} = \nabla \times \vec{A}_o$. Moreover, $\vec{A}_o = 0$ for $r >> 0$.

**(a.)** let $\lambda$ be scalar function, show $\vec{A} = \vec{A}_o + \nabla\lambda$ is a vector potential for $\vec{B}$

**(b.)** show $\nabla \bullet \vec{A} = 0$ implies $\lambda$ solves a Poisson equation where $\nabla \bullet \vec{A}_o$ is playing the role of charge density. By analogy with the electrostatics theorem:

*If charge density is $\rho$ then the potential is* $V(\vec{r}) = \frac{1}{4\pi\epsilon_o}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}} d\tau'$

write down the integral formula for $\lambda$

**(c.)** given $\vec{A}$ satisfies the Coulomb Gauge, show that $\nabla^2 \vec{A} = -\mu_o \vec{J}$.

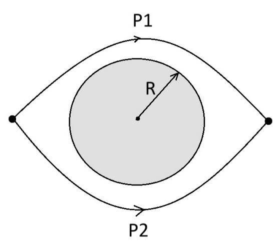
**(d.)** explain why $\vec{A}(\vec{r}\,) = \dfrac{\mu_o}{4\pi}\displaystyle\int \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}} d\tau'$ where $\mathscr{r} = \|\vec{r} - \vec{r}\,'\,\|$

**Problem 2** (30pts) In quantum mechanics the phase of a charge $q$ moving along a path with vector potential $\vec{A}$ is given a phase shift $\varphi = \frac{q}{\hbar}\int_P \vec{A} \bullet d\vec{l}$.

**(a.)** Consider a very long solenoid of radius $R$ centered about the $z$-axis. A current $I$ flows through the solenoid which has $n$ turns per unit length. Find the vector potential for the magnetic field for $0 \leq s \leq R$.

**(b.)** Find the vector potential outside the solenoid where $s > R$.

**(c.)** Let $P_1$ and $P_2$ be two coterminal paths around the solenoid; that is suppose $P_1, P_2$ are paths for which $P_1 \cup (-P_2)$ then forms a loop enclosing around the solenoid. Let $\varphi_j$ be the phase shift along $P_j$. Find explicitly how the difference in phase shift $\varphi_2 - \varphi_1$ depends on the current $I$.



**Problem 3** (20pts) A very long circular cylinder of radius $R$ carries a magnetization $\vec{M} = ks^2\widehat{\phi}$ where $k$ is constant. Find the magnetic field for points inside and outside $s = R$ as follows:

**(a.)** solve via the $\vec{H}$ arguments

**(b.)** solve via analysis of Ampere's Law on the bound currents

**Problem 4** (20pts) A square loop of side length $a$ lies in the $xy$-plane in the first quadrant with one corner at $(0, 0)$. In this region $\vec{B} = 3ky^2t^2\hat{z}$ where $k$ is a constant. Calculate the emf in the loop.

**Problem 5** (5pts) We introduced the stress energy tensor and we found the formula

$$\vec{F} = \int_{\partial V} T(d\vec{a},\ ) - \mu_o \epsilon_o \frac{d}{dt} \int_V \vec{S} d\tau.$$

Give a qualitative description of what this formula tells us.

**Problem 6** (5pts) We defined $\vec{g} = \mu_o \epsilon_o \vec{S}$ and we showed

$$-\frac{\partial \vec{g}}{\partial t} + \sum_{i,j=1}^{3} (\partial_i T_{ij}) \hat{x}_j = 0.$$

Qualitatively describe the significance of the above equation.

**Problem 7** (20pts) Notice for part (b.) We showed in lecture that the magnetic field within a spinning charged spherical shell with uniform charge density $\sigma$ at radius $R$ with angular velocity $\vec{\omega}$ is given by $\vec{B} = \frac{2}{3}\mu_o \sigma R \vec{\omega}$.

**(a.)** Suppose a solid sphere of radius $R$ has uniform magnetization $\vec{M} = M\hat{z}$ where $M > 0$. Calculate the bound current densities inside the sphere and on its surface

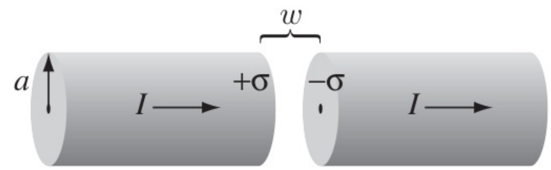
**(b.)** Calculate the magnetic field for the uniformly magnetized sphere in the case $r < R$ (inside the sphere). Your solution should not use $\vec{H}$.

**(c.)** Derive the magnetic field within the uniformly magnetized sphere via auxillary field arguments in the case $r < R$. (now use $\vec{H}$)

**(d.)** Find the magnetic field for the uniformly magnetized sphere in the case $r > R$.

**Problem 8** (20pts) We argued that the rate of work done on charges due to the electric and magnetic fields in some volume $V$ is given by $\frac{dW}{dt} = \int_V \vec{E} \bullet \vec{J} d\tau$. Show that $\frac{dW}{dt} = -\frac{d}{dt}\int_V u\, d\tau - \int_{\partial S} \vec{S} \bullet d\vec{A}$ where $u$ is the electromagnetic field energy density and $\vec{S}$ is the Poynting vector.

**Problem 9** (20pts) Consider the model of the charging capacitor pictured below.




**(a.)** Find $\vec{E}$ near the center of the gap assuming that the charging begins at $t = 0$

**(b.)** Find $\vec{B}$ due to the changing electric field near the center of the gap

**(c.)** Find the electromagnetic field energy density $u$ and Poynting vector $\vec{S}$ near the center of the gap of width $w$

**(d.)** Do $u$ and $\vec{S}$ relate in the appropriate manner ?