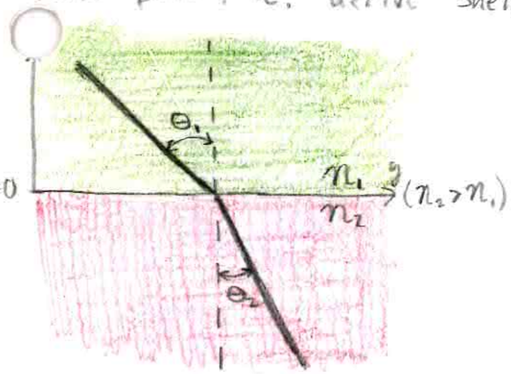
$\boxed{6-7}$ Consider light passing from medium with index of refraction n_1 into another medium of index of refraction n_2. Use Fermat's principle to find least time path, ie. derive Snell's Law $n_1 \sin\theta_1 = n_2 \sin\theta_2$.



$n = c/v$ thus $v_1 = c/n_1$
$v_2 = c/n_2$

c = speed of light in vaccum

$$t = \int \frac{ds}{v(x)} = \int \frac{dx^2 + dy^2}{v(x)} = \int \frac{\sqrt{dx^2+dy^2}}{v(x)} = \int \frac{\sqrt{1+y'^2}}{v(x)}\,dx \qquad y' = \frac{dy}{dx}$$

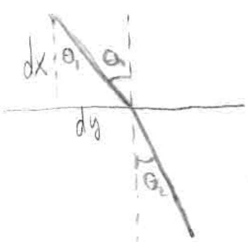
the function $f = \frac{\sqrt{1+y'^2}}{v}$ then $\frac{\partial f}{\partial y} = 0$ and $\frac{\partial f}{\partial y'} = \frac{y'}{v\sqrt{1+y'^2}}$

Euler-Lagrange yields minimum functional with $\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial y} = 0$

$\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 \Rightarrow \frac{\partial f}{\partial y'}$ = const. with respect to x $= a = \frac{y'}{v\sqrt{1+y'^2}}$

$$a = \frac{y'}{v\sqrt{1+y'^2}} = \frac{-\tan\theta}{v\sqrt{1+\tan^2\theta}} = \frac{-\tan\theta}{v\sqrt{\sec^2\theta}} = \frac{-\sin\theta}{\cos\theta}\left(\frac{1}{\frac{v}{\cos\theta}}\right) = -\sin\theta$$

$$-a = \frac{\sin\theta}{v} = \frac{n\sin\theta}{c}$$



$\Rightarrow \tan\theta = \frac{dx}{-dy}$

$\Rightarrow \tan\theta = \frac{-dx}{dy}$

$\Rightarrow$ const $= n\sin\theta$

and n could well have been a function of x. That is $n = n(x)$ which could be used to describe the gradual bending of starlight by the differing types of air at different altitudes

In general then $\boxed{n(x)\sin\theta = \text{const.}}$

specifically if $n(x) = \begin{Bmatrix} n_1 & \text{for} & x \geq 0 \\ n_2 & \text{for} & x < 0 \end{Bmatrix}$ then $\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$ ✓