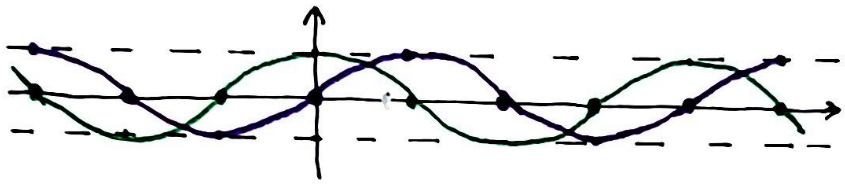
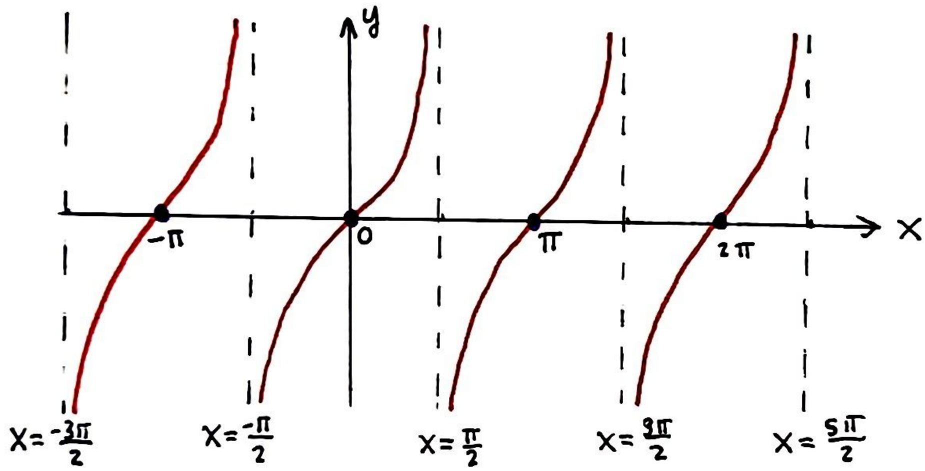
GRAPHS OF OTHER TRIGONOMETRIC FUNCTIONS

Analyzing graph of $y = \tan(x)$

To graph $y = \tan(x) = \dfrac{\sin(x)}{\cos(x)}$ we should focus on which $x \in \mathbb{R}$ cause $\cos(x) = 0$. and $\sin(x) = 0$

these x are where tangent is zero

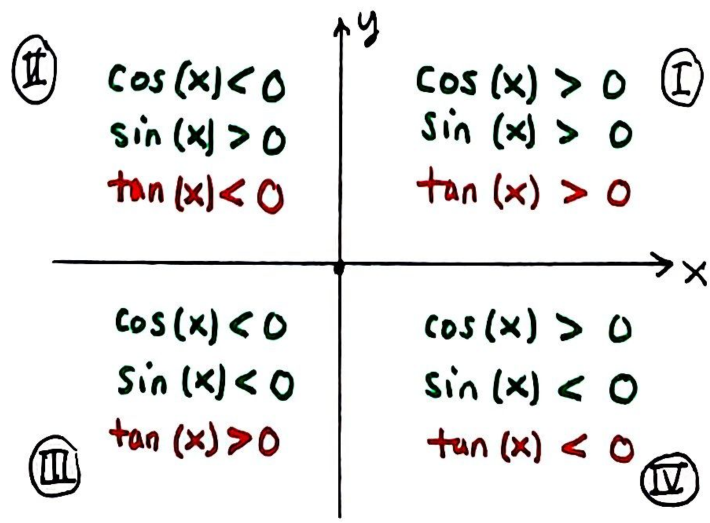




If $f(x) = \tan(x)$ then the PERIOD OF tangent is π

$f(x) = f(x + n\pi)$ for all $n \in \mathbb{Z}$

Remark: since $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$ if both $\sin(x)$ and $\cos(x)$ are positive (or negative) then $\tan(x) > 0$ whereas if $\sin(x)\cos(x) = \pm$ then $\tan(x) < 0$
We can understand this by a quadrant picture



Modified Graphs:

Given $y = f(x)$ recall that: (assume $c > 0$)

1.) $y = f(x) + c$	shifts vertically c-units
2.) $y = f(x-c)$	shifts right c-units
3.) $y = c\,f(x)$	stretches vertically by c-scale
4.) $y = f(cx)$	compresses horizontal features by c-factor (or stretches by $1/c$ factor)
5.) $y = -f(x)$	flips graph over x-axis
6.) $y = f(-x)$	flips graph over y-axis

These can be combined of course. ALL the Th$^{\text{m}}$ boxes in § 6.2 are simply these steps applied to trig. functions.

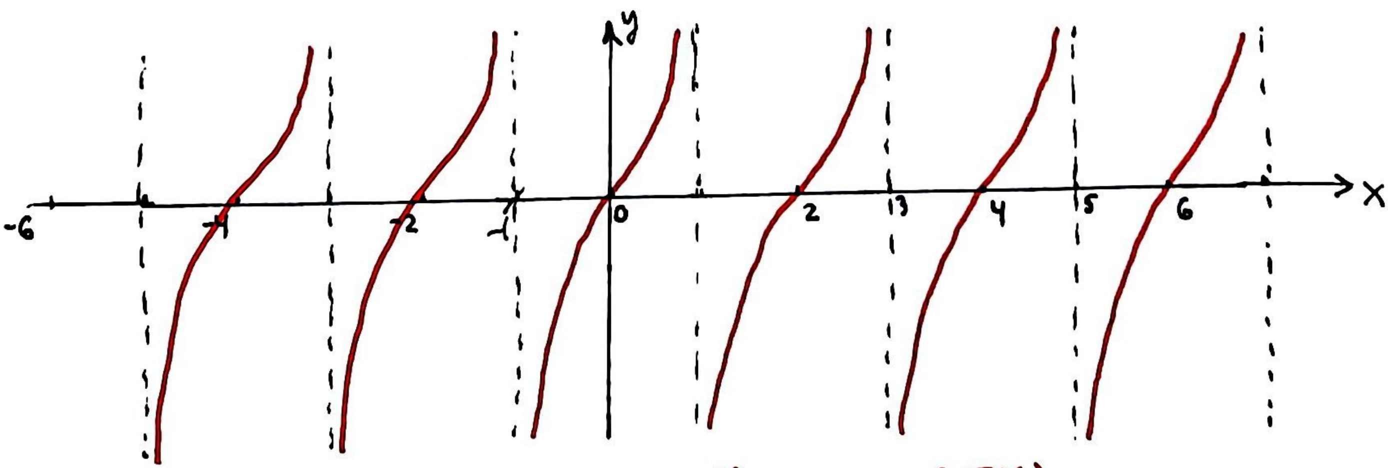
Example: $y = 3\tan\left(\frac{\pi x}{2}\right)$

$\uparrow$ stretches vertically

$\uparrow$ $c = \pi/2 \rightsquigarrow$ stretches by $\frac{2}{\pi}$

ZEROES: $\frac{2}{\pi}\{-\pi, 0, \pi, 2\pi, \dots\} = \{-2, 0, 2, 4, \dots\}$

V.A.'s: $\frac{2}{\pi}\left\{-\frac{3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}\right\} = \{-3, -1, 1, 3, 5, \dots\}$



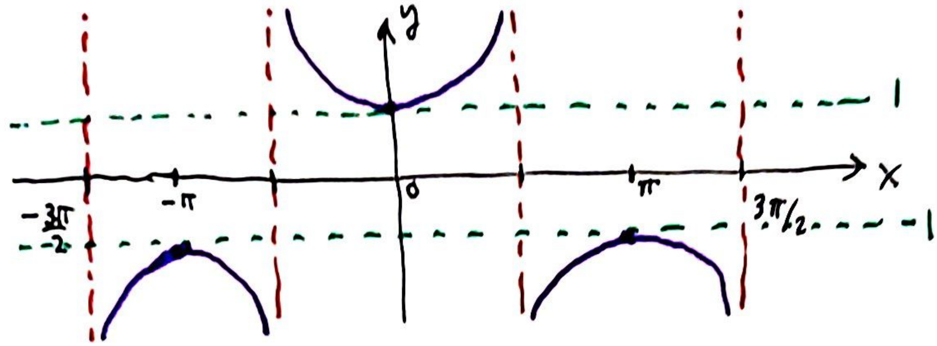
$y = 3\tan\left(\frac{\pi x}{2}\right)$

<u>Example</u>: $y = \sec(x) = \dfrac{1}{\cos(x)}$

Since $|\cos(x)| \leq 1$

note $\dfrac{1}{|\cos(x)|} \geq 1$

the graph of secant is stuck outside $|y| \geq 1$



$$\text{range}(\sec(x)) = (-\infty, -1] \cup [1, \infty) = \text{range}(\csc(x))$$

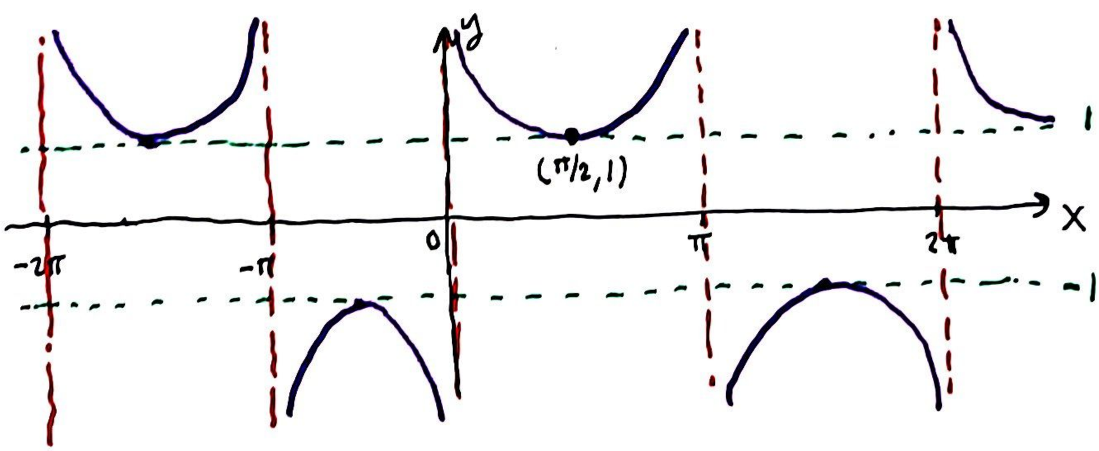
<u>Example</u>: $y = \csc(x) = \dfrac{1}{\sin(x)}$: again $|\sin(x)| \leq 1$ gives $\dfrac{1}{|\sin x|} \geq 1$ thus $y = \dfrac{1}{\sin(x)}$ is stuck outside $|y| \leq 1$.

has vertical asymptotes where $\sin(x) = 0$. Recall we learned

$\sin(x) = 0 \iff x = n\pi$ for $n \in \mathbb{Z}$

$\iff x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

$\sin\left(\frac{\pi}{2}\right) = 1$



REMARK: I WANT YOU TO <u>UNDERSTAND</u> WHERE ZEROS and VERTICAL ASMPTOTES ARISE FROM FOR TRIG. GRAPHS. THE REST OF THIS STRETCHING / COMPRESS SHIFT, YOU CAN HELP YOURSELF OUT WITH GRAPHING CALCULATOR ETC. OF COURSE THIS CAN <u>ALL</u> BE DONE BY HAND. PLEASE READ §6.2 FOR MORE.

§ 6.3 INVERSE TRIGONOMETRIC FUNCTIONS

Def^{n}/ If $f : A \to B$ is an 1-1 function then $f^{-1} : B \to A$ is the <u>inverse function</u> of f and we <u>define</u> $f^{-1}(y) = x$ if and only if $f(x) = y$. (the inverse function undoes the function)

<u>Alternatively</u>,

$$f(f^{-1}(y)) = y \quad \text{for all } y \in B$$
$$f^{-1}(f(x)) = x \quad \text{for all } x \in A$$

Notice $\text{domain}(f) = \text{range}(f^{-1}) = A$ and $\text{domain}(f^{-1}) = \text{range}(f) = B$.

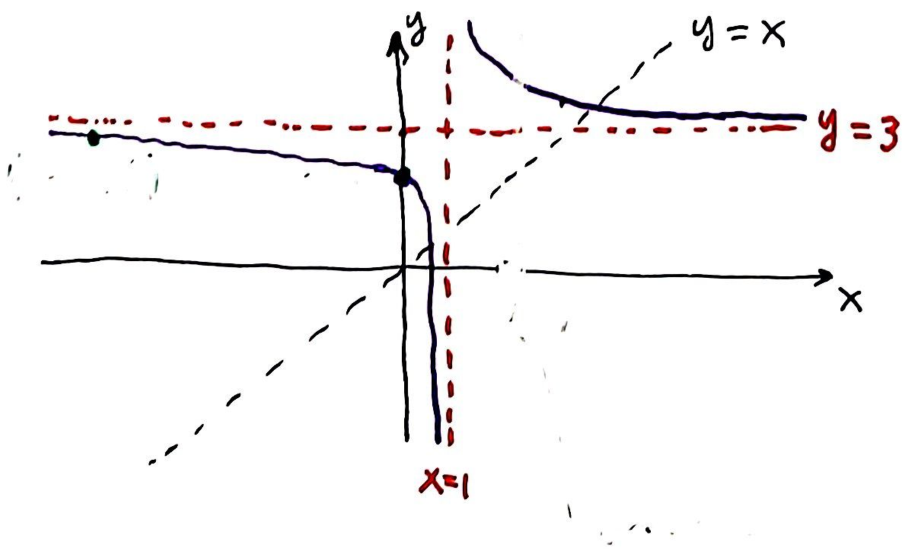
<u>Example</u>: $f(x) = \dfrac{3x-2}{x-1}$ has $\underline{\text{dom}(f) = \mathbb{R} - \{1\} = \text{range}(f^{-1})}$

to calculate $f^{-1}(y)$ simply solve $y = \dfrac{3x-2}{x-1}$ for x

$$(x-1)y = 3x-2$$
$$xy - y = 3x - 2$$
$$xy - 3x = y - 2$$
$$x(y-3) = y-2$$
$$x = \frac{y-2}{y-3} \quad \therefore \quad \boxed{f^{-1}(y) = \frac{y-2}{y-3}}$$

$\dfrac{y-3+1}{y-3} = 1 + \dfrac{1}{y-3}$

$\Leftrightarrow$ $\underline{\text{dom}(f^{-1}) = \mathbb{R} - \{3\}}$ $= \underline{\text{range}(f)}$



$y = \dfrac{3x-2}{x-1}$

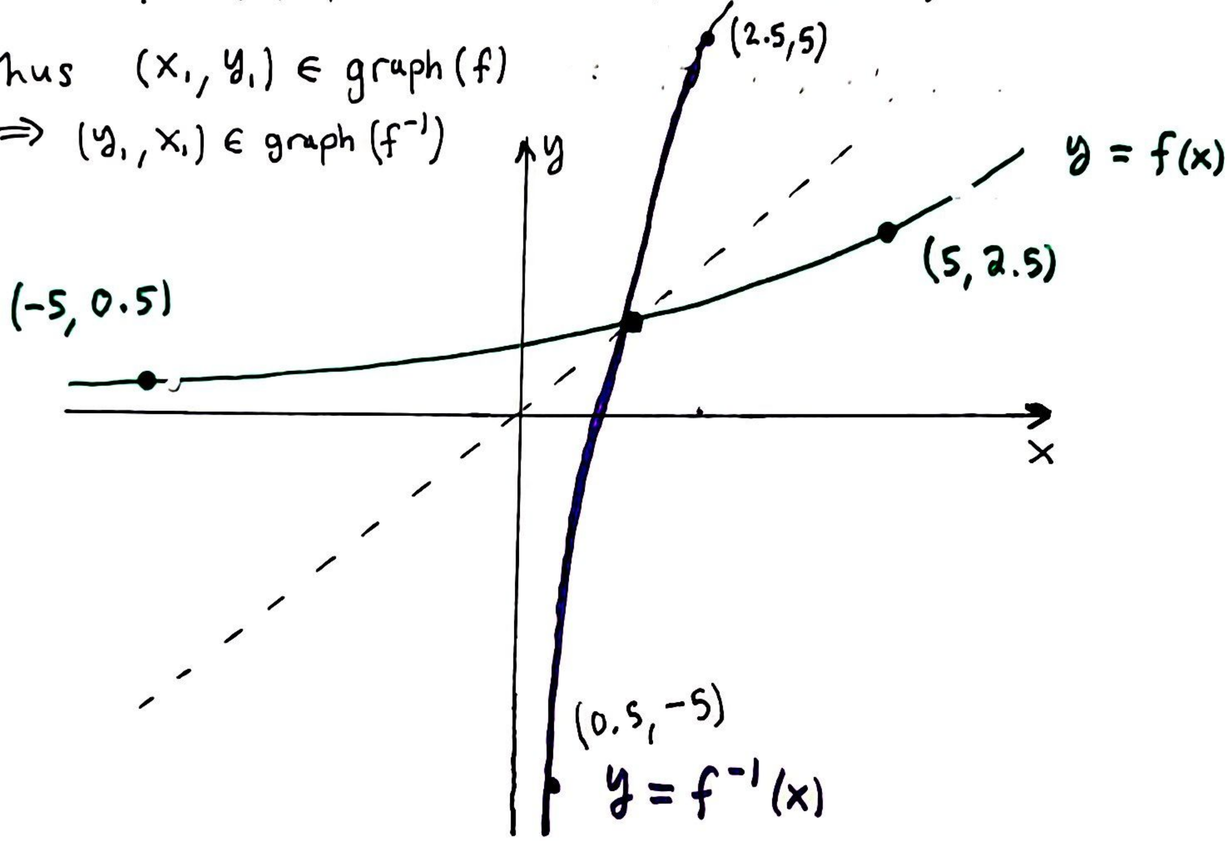
<u>Remark</u>: my bad! I tried to graph $y = f^{-1}(x)$ here, but it's just ugly. I give easier example

Example: how $y = f(x)$ and $y = f^{-1}(x)$ relate

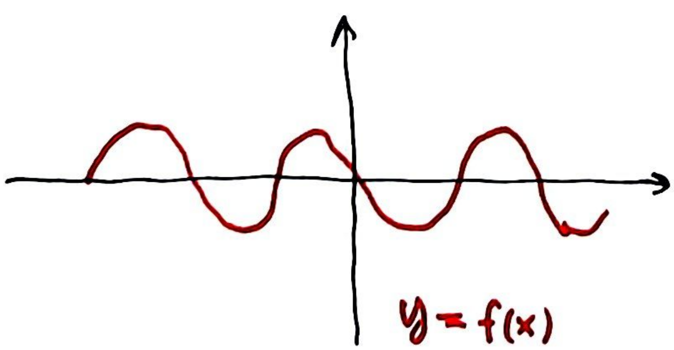
If $y_1 = f(x_1)$ then $f^{-1}(y_1) = f^{-1}(f(x_1)) = x_1$

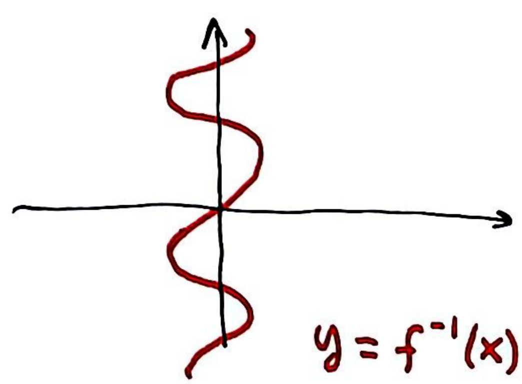
thus $(x_1, y_1) \in \text{graph}(f)$

$\Rightarrow (y_1, x_1) \in \text{graph}(f^{-1})$



REMARK: TO GRAPH $y = f^{-1}(x)$ we simply graph $y = f(x)$ but swap the roles of x and y.





OH NOES!

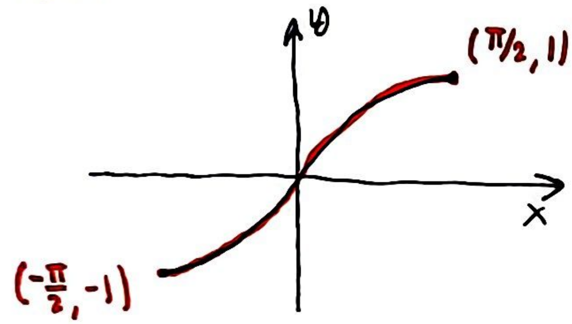
VERTICAL LINE TEST FAIL!!!

(WHAT TO DO?)

LOCAL INVERSES

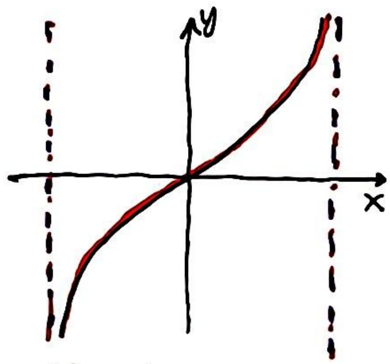
When we talk about $\sin^{-1}(x)$, $\cos^{-1}(x)$ and $\tan^{-1}(x)$ we are <u>not</u> talking about a <u>global</u> inverse for sine, cosine and tangent. There is no global inverse because sine, cosine and tangent repeat. We need to select subgraphs of $y = \sin x$, $\cos x$ and $\tan(x)$ for which the graphs pass the <u>horizontal</u> line test.

CUSTOM: USE THESE SUBGRAPHS:



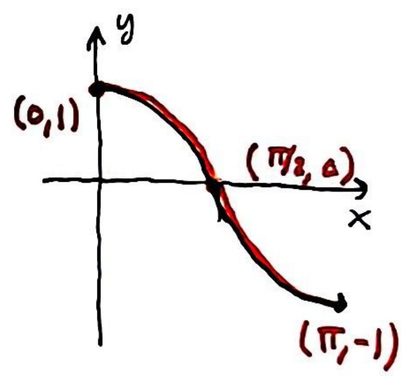
$y = \sin(x)$

$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$



$y = \tan(x)$

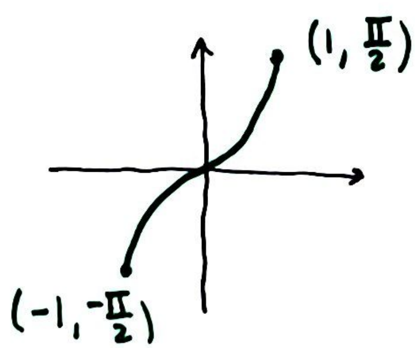
$-\frac{\pi}{2} < x < \frac{\pi}{2}$



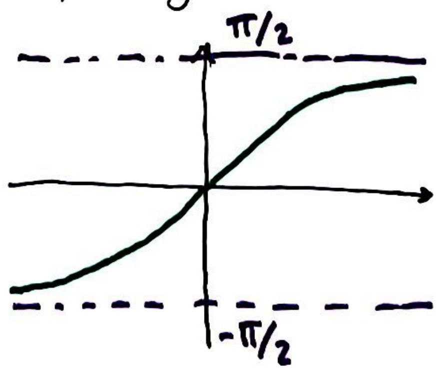
$y = \cos(x)$

$0 \leq x \leq \pi$

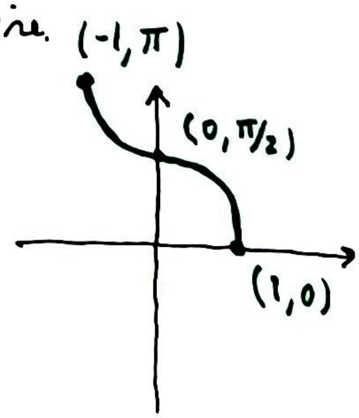
- inverse trig. functions are inverses of the above <u>restrictions</u> of the functions sine, tangent and cosine.



$y = \sin^{-1}(x)$



$y = \tan^{-1}(x)$



$y = \cos^{-1}(x)$

Def^n/ $\sin(\sin^{-1}(x)) = x$ for $-1 \le x \le 1$

$\sin^{-1}(\sin(y)) = y$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

Example:

$\sin^{-1}(0.5) = \frac{\pi}{6}$ since $\sin\left(\frac{\pi}{6}\right) = 0.5.$

$\sin^{-1}(2) = \text{ERROR}$

Def^n/ $\cos(\cos^{-1}(x)) = x$ for $-1 \le x \le 1$

$\cos^{-1}(\cos(y)) = y$ for $0 \le y \le \pi$

Example:

$\cos^{-1}(0.5) = \frac{\pi}{3}$ since $\cos\left(\frac{\pi}{3}\right) = 0.5$

$\cos^{-1}(3) = \text{ERROR}$

$\cos(-\pi) = -1$ then $\underbrace{\underbrace{\cos^{-1}(\cos(-\pi))}_{-\pi}}_{\text{BOGUS!}} = \underbrace{\cos^{-1}(-1)}_{\pi}$

$\cos^{-1}(\cos(y)) = y$

ONLY for $0 \le y \le \pi$

Def^n/ $\tan(\tan^{-1}(x)) = x$ for $x \in \mathbb{R}$

$\tan^{-1}(\tan(y)) = y$ for $\frac{-\pi}{2} < y < \frac{\pi}{2}$

Example:

$$\tan^{-1}(2) \cong 1.107149\ldots$$

$$\tan(1.107149\ldots) = 2$$

$$\underbrace{\tan(\pi) = 0}$$

take $\tan^{-1}$ of formula

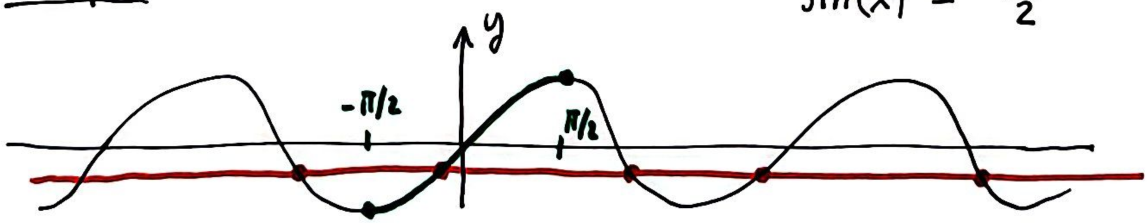
$$\cancel{\underbrace{\tan^{-1}(\tan \pi)}_{\pi}} = \underbrace{\tan^{-1}(0)}_{0}$$

$$\cancel{\pi} = 0$$

WHAT!

Example

$\sin(x) = \frac{-1}{2}$



$$\sin^{-1}(\sin(x)) = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$x = \sin^{-1}\left(-\frac{1}{2}\right) = \frac{-\pi}{6}.$$