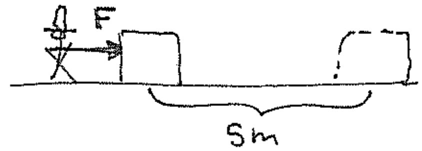
— WORK AND ENERGY —

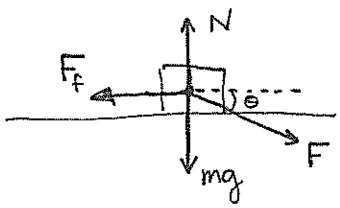
$\boxed{E1}$ If we push a 100 kg box across a rough floor with $\mu_k = 0.5$ such that we maintain a constant speed over the distance 5m then the <u>work</u> done by us is

$$
\begin{aligned}
W &= (5m)(\text{force applied}) \\
&= (5m)(0.5)(100kg)(9.8\,\tfrac{m}{s^2}) \\
&= 2450\ \frac{kgm}{s^2}
\end{aligned}
$$



$F = \mu_k N = \mu_k mg$
applied <u>horizontally</u>.

$\boxed{E2}$ Again, same box, same distance, same floor except, push at angle θ above horizontal such that get constant speed.



$$N - mg - F\sin\theta = ma_y = 0$$
$$F\cos\theta - F_f = ma_x = 0$$

Note, $N = mg + F\sin\theta \Rightarrow F_f = \mu N = \mu(mg + F\sin\theta)$

Substituting into 2^{nd} Eq^{n} yields: $F\cos\theta - \mu(mg + F\sin\theta) = 0$

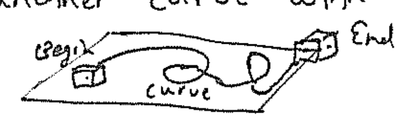
Hence $F(\cos\theta + \mu\sin\theta) = \mu mg \;\therefore\; F = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$.

Only $F\cos\theta = F_x$ does work here,

$$
\begin{aligned}
W &= (5m)(\text{force applied}) \\
&= (5m)\left(\frac{(0.5)(100kg)(9.8\,m/s^2)}{\cos\theta + \mu\sin\theta}\right)\cos\theta
\end{aligned}
$$

when $\theta = 0°$ we recover $\boxed{E1}$.

$\boxed{E3}$ Take same box and move it with a horizontally applied force F_o over another curve with arclength l then $W = F_o l$.



$\boxed{E4}$ If we push the box with additional force then the box <u>accelerates</u> and we increase the <u>kinetic</u> <u>energy</u> of the box.

Work - Energy - Thm

$\boxed{\text{Def}^{\underline{n}}/ \quad K = \frac{1}{2}m(\vec{v}\cdot\vec{v}) \text{ for } m \text{ at } \vec{r}(t), \text{ where we define } \vec{v} = d\vec{r}/dt.}$

Case of zero force:

$$m\vec{a} = 0 \longrightarrow m\frac{d\vec{v}}{dt} = 0$$

Note,
$$\frac{d}{dt}(K) = \frac{d}{dt}\left(\frac{1}{2}m\vec{v}\cdot\vec{v}\right) = \frac{1}{2}m\left(\frac{d\vec{v}}{dt}\cdot\vec{v} + \vec{v}\cdot\frac{d\vec{v}}{dt}\right)$$
$$= m\vec{v}\cdot\frac{d\vec{v}}{dt}$$
$$= \vec{v}\cdot\left(m\frac{d\vec{v}}{dt}\right)$$
$$= \vec{v}\cdot\vec{F}$$
$$= 0. \quad \therefore \underline{K \text{ constant}}.$$

<u>Note</u>,: $\underbrace{\int_{t_i}^{t_f}\frac{dK}{dt}\,dt}_{} = \int_{t_i}^{t_f}\vec{v}(t)\cdot\vec{F}(\vec{r}(t))\,dt$ (using above calculation where $\vec{F}$ need not be zero)

$$K(t_f) - K(t_i) = \int_C \vec{F}\cdot d\vec{r}$$

where C goes from $\vec{r}(t_i)$ to $\vec{r}(t_f)$) gives work done by $\vec{F}$.

$\boxed{\underline{\text{Concept}}: \ \Delta KE = \text{work done by } \vec{F}_{net}.}$

$\boxed{\text{Def}^{\underline{n}}/ \text{ If } \vec{F} \text{ is a } \underline{\text{conservative}} \text{ force then } \vec{F} = -\nabla U = \left\langle -\frac{\partial u}{\partial x}, -\frac{\partial u}{\partial y}, -\frac{\partial u}{\partial z}\right\rangle \text{ where } U \text{ is the } \underline{\text{potential energy}} \text{ function for } \vec{F}.}$

<u>Again</u>:
$$K(t_f) - K(t_i) = \int_C \vec{F}\cdot d\vec{r}$$
$$= -\int_C \nabla U\cdot d\vec{r}$$
$$= -(U_f - U_i) \longrightarrow \boxed{K_f + U_f = K_i + U_i}$$

$\therefore$ $\boxed{E = K + U \text{ is conserved}; \quad E_i = E_f \text{ or } E(t_i) = E(t_f) \ \forall t_i, t_f}$

Conservation of total energy!

Def$^{\underline{n}}$/ If a curve C has parametrization $\vec{r}: [a,b] \longrightarrow \mathbb{R}^3$ so $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ then the line integral of a force $\vec{F}$ is defined,

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \left(\vec{F}(\vec{r}(t)) \cdot \frac{d\vec{r}}{dt} \right) dt$$

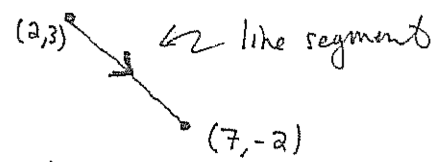
physically this gives the <u>work</u> done by $\vec{F}$ to move some object along C.

<u>Example</u>: Suppose $\vec{F}(x,y) = x^2 \hat{i} + y \hat{j} = \langle x^2, y \rangle$ (omitting units for this example)

Let C be the line segment from $(2,3)$ to $(7,-2)$. We can construct

$$\vec{r}(t) = (2,3) + t\big((7,-2) - (2,3) \big)$$

$$\left.\begin{array}{l} \Rightarrow \vec{r}(t) = \langle 2+5t,\ 3+5t \rangle \\ \text{or} \quad \underbrace{x = 2+5t, \quad y = 3+5t}_{\text{this pair of scalar eq}^{\underline{n}}\text{'s is summarized by the single vector } \vec{r}(t).} \end{array}\right\} \quad 0 \leq t \leq 1.$$



Now calculate,

$$W = \int_C \vec{F} \cdot d\vec{r} = \int_0^1 \left(\vec{F}(2+5t, 3+5t) \cdot \frac{d}{dt}\big(\langle 2+5t, 3+5t \rangle\big) \right) dt$$

$$= \int_0^1 \langle (2+5t)^2, (3+5t) \rangle \cdot \langle 5, 5 \rangle \, dt$$

$$= \int_0^1 \left[5(2+5t)^2 + 5(3+5t) \right] dt$$

$$= \left[\frac{1}{3}(2+5t)^3 + \frac{1}{2}(3+5t)^2 \right]_0^1$$

$$= \frac{1}{3}(7^3 - 2^3) + \frac{1}{2}(8^2 - 3^2)$$

$$= \frac{1}{3}(243 - 8) + \frac{1}{2}(64 - 9)$$