

Don't even think about working these problems out on this page alone. The solutions should be written neatly on lined or unlined paper with the work clearly labeled. Do not omit scratch work. I need to see all steps. Thanks and enjoy. Notice that the A,B,C labels refer to the system described in the syllabus. I expect everyone to do the type-A problems however you may pick and choose from the type-B or C problems. This is due 3-11-2010 at start of lecture.

Problem A27 [35pts] Edwards #6.9, 6.10 from pg. 128.

Problem A28 [35pts] Edwards #7.3, 7.4 from pg. 141.

Problem A28 [35pts] Edwards #7.10, 7.12 from pg. 141.

Problem A29 [35pts] Edwards #8.3 from pg. 158.

Problem A30 [35pts] Edwards #3.1 from pg. 194.

Problem A31 [35pts] Edwards #3.5, 3.6 from pg. 194.

Problem A32 [35pts] Edwards #3.9 from pg. 195 (use the iterative formula which justified the implicit function theorem).

this concludes part A of Problem Set III, also I said you could turn in Problem A22 with Problem Set IIIb on the Friday after Easter break.

Problem A33 [35pts] Edwards #1.1, 1.8 from pg. 171 (gifts I think).

Problem A34 [35pts] Find a coordinate chart for the mobius strip. In other words, find an inverse for the coordinate patch I gave in the notes. You may shrink the domain of the coordinate patch if need be.

Problem A35 [35pts] Use Example 6.2.9 to find a 3-dimensional manifold in \mathbb{R}^3

Problem A36 [35pts] Find a connected subset of \mathbb{R}^3 which can be viewed as a manifold with coordinate chart F from Example 6.2.10.

Problem A37 [35pts] The inertia tensor $[I_{ij}]$ for a rigid object B provides nice formulas to describe the possible rotational motions of the rigid body. In particular, we define,

$$I_{ij} = \iiint_B \rho [(x_1^2 + x_2^2 + x_3^2)\delta_{ij} - x_i x_j] dV$$

It can be shown that if the body B rotates with angular velocity $\vec{\omega}$ then the total kinetic energy of B is given by

$$T = \sum_{i,j=1}^3 I_{ij} \omega_i \omega_j = \vec{\omega}^T I \vec{\omega}.$$

The total angular momentum of B is given by

$$\vec{L} = \sum_{i,j=1}^3 I_{ij} \omega_i e_j = I \vec{\omega}.$$

Finally, the net torque on B governs the change in the angular momentum:

$$\vec{\tau} = \frac{d\vec{L}}{dt} = I \frac{d\vec{\omega}}{dt}$$

Show that if $\vec{\tau} = 0$ then the kinetic energy is conserved. In other words, show that if $\vec{\tau} = 0$ then $\frac{dT}{dt} = 0$. warning: if your solution is longer than about a line then you're not thinking about this in the best way.

Problem A38 [35pts] Consider the inertia tensor below:

$$[I_{ij}] = \frac{Ma^2}{12} \begin{bmatrix} 8 & -3 & -3 \\ -3 & 8 & -3 \\ -3 & -3 & 8 \end{bmatrix}$$

This is the inertia tensor for a cube of side length a with one corner at the origin. The total mass M of the cube is distributed evenly to give $\rho = M/a^3$.

Find the principle moments of inertia (these are the eigenvalues of I). Then find the principle axes of of inertia (these are the eigenvectors of I). Describe the motion of the system if $\vec{\omega}_o$ is paralell to a principle axis. What is the difference if the initial angular velocity vector is not in the direction of an axis of symmetry ? You can assume the net torque is zero hence $d\vec{L}/dt = 0$. Moreover, as $\vec{L} = I\vec{\omega}$ we have $I\omega_f = I\omega_o$.

Problem B3 [100pts] Edwards #7.1, 7.2 from pg. 140.

Problem C5 [100pts] animate motion of spinning top etc...

Problem C6 [100pts] find critical points, overlay quadratic form near point