

Don't even think about working these problems out on this page alone. The solutions should be written neatly on lined or unlined paper with the work clearly labeled. Do not omit scratch work. I need to see all steps. Thanks and enjoy. Notice that the A,B,C labels refer to the system described in the syllabus. I expect everyone to do the type-A problems however you may pick and choose from the type-B or C problems. This is due 2-16-2010 at start of lecture.

Problem A13 [35pts] Let $\gamma(t) = \langle \cos(t), t, \sin(t) \rangle$. Calculate T, N, B and the curvature and torsion.

Problem A14 [35pts] Edwards 1.10 and 1.11 from pg. 63 (these are fun and very applicable to physics of motion).

Problem A15 [25pts] Use the notation of the Euclidean geometry and physics chapter to derive the coriolis effect (you all should make me do most of this problem in lecture at some point !)

Problem A16 [80pts] Calculate the derivative for each of the mappings below:

1. $F(x, y, z) = x^2 + y^2 + z^2$
2. $\gamma(t) = (t, t^2, t^3)$
3. $F \circ \gamma$
4. $f(x, y) = (x, y, 1/x + 1/y)$
5. $g(s, t) = (st, s + t)$
6. $f \circ g$
7. $h(r, \theta, z) = (r \cos \theta, r \sin \theta, z)$
8. $X(s, t) = (R \cos s \sin t, R \sin s \cos t, R \cos t)$ for a constant $R > 0$.

Problem A17 [35pts] Given the mappings f and X from the preceding problem, describe domains $U = \text{dom}(f)$ and $V = \text{dom}(X)$ for f and X which make $f(U)$ and $X(V)$ patched manifolds of dimension 2 in \mathbb{R}^3 (or in plain english, smooth surfaces in \mathbb{R}^3).

Problem A18 [35pts] Prove that $\frac{\partial x_i}{\partial x_j} = \delta_{ij}$ by explicitly calculating the partial derivative of $f : \mathbb{R}^n \rightarrow \mathbb{R}$ defined by $f(x) = x_i$ with respect to x_j . In other notation, calculate $D_j f$ directly from the definition of partial derivatives.

Problem A19 [35pts] Let $f(x, y) = x^y$ and calculate the differential df . Let $\gamma(t) = (t, t)$ and use the chain rule to calculate $(f \circ \gamma)'(t)$ (*note the chain rule is accomplished via multiplication of Jacobian matrices $[df]$ and $[\gamma]$ in this context*). Contrast this calculation to the calculation you used in calculus I to find $\frac{d}{dx}(x^x)$.

Problem A20 [35pts] Edwards #2.7 from pg. 75.

Problem A21 [35pts] Edwards #3.13 from pg. 89.

Problem A22 [35pts] Edwards #3.12, 3.17 from pg. 195.

Problem A23 [35pts] Edwards #4.1, 4.2, 4.3 from pg. 99.

Problem A24 [35pts] Edwards #4.10, 4.12 from pg. 99.

Problem A25 [100pts] Edwards #5.4, 5.5, 5.6 from pg. 116.

Problem A26 [100pts] Edwards #5.10, 5.11, 5.12, 5.14 from pg. 116-117.

Problem B3 [100pts] (moved to next Problem Set)

Problem C3 [100pts] Let $\gamma(t) = \langle t, t^2, t^3 \rangle$. Calculate T, N, B and the curvature and torsion.
Animate the TNB frame moving along the path.

Problem C4 [100pts] Graph parametrized surfaces including tracelines to indicate the coordinate curves.