Don't even think about working these problems out on this page alone. The solutions should be written neatly on lined or unlined paper with the work clearly labeled. Do not omit scratch work. I need to see all steps. Thanks and enjoy. Notice that the A,B,C labels refer to the system described in the syllabus. I expect everyone to do the type-A problems however you may pick and choose from the type-B or C problems. This is due 2-11-2010 at start of lecture.

Problem A1 [35pts] Let $v =< 1, 2, 3 >$ and $w =< c, 4, 5 >$ where $c \in \mathbb{R}$,

1. calculate $v + w$
2. calculate $v \cdot w$
3. find $||v||$ and $||w||$
4. what choice of c makes v orthogonal to w
5. what value of c makes the angle between v and w equal to θ

Yes, your answers for (1-3) have a c in them and (5) has a θ.

Problem A2 [35pts] Supply the implied domains and codomains for mappings with the formulas given below. Also, for (5),(6) and (7) find their formula.

1. $G(x, y, z) = (x^2, \sqrt{x-y}, \frac{1}{z})$
2. $L(x, y) = (x + y, x - y)$
3. $f(x, y, z) = (x, z)$
4. $T(x, y, z) = (x + y, z - y, x)$
5. $G + T$
6. $f \circ \xi + L$ where $\xi(x, y) = (x, y, 0)$
7. $L \circ f$

Problem A3 [35pts] State which mappings in the previous problem are linear. Then prove their linearity by find a matrix which induces the mapping; that is find a matrix A such that $F(x) = Ax$ for the mapping F.

Problem A4 [45pts] An unusual coordinate system on $\mathbb{R}^2$ is given by the formulas below:

$$x = r\cosh(\phi) \qquad y = r\sinh(\phi)$$

Invert these equations to find $r = r(x, y)$ and $\phi = \phi(x, y)$. Describe some subset of $\mathbb{R}^2$ for which the mapping $\Phi(r, \phi) = (r\cosh(\phi), r\sinh(\phi))$. Graph the coordinate curves $\phi = c_1$ and $r = c_2$ for a few values of c_1, c_2. Is this an orthogonal coordinate system? Is it curvelinear or rectilinear?

Problem A5 [35pts] Level curves can be understood in terms of inverse images. Describe the curves or surfaces given by the inverse images below: if it's not much trouble then illustrate your answer with a simple hand-drawn graph (or use Mathematica if you prefer). Assume that $a, b, c, d \in \mathbb{R}$.

1. let $F(x,y,z) = x^2 + y^2 + z^2$ what is $F^{-1}(\{R\})$ for $R > 0$. What is $F^{-1}(\{0\})$?
2. let $G(x,y,z) = ax + bx + cz + d$ what is $G^{-1}(\{0\})$?
3. let $H(x,y) = x^2 - y$ what is $H^{-1}(\{b\})$ for $b \in \mathbb{R}$?
4. let $f(x,y) = x^2/a^2 + y^2/b^2$ what is $f^{-1}(\{1\})$?
5. let $\eta(t,x,y) = -t^2 + x^2 + y^2$ what is $\eta^{-1}(\{0\})$?

Problem A6 [35pts] Give parametrizations of the curves, surfaces and volume described in the preceding problem. In each case clearly describe the domain of the mapping which defines the parametrization. Remember that $X : U \to \mathbb{R}^n$ gives a parametrization of $S \subseteq \mathbb{R}^n$ if X is a mostly one-one mapping such that $X(U) = S$.

Problem A7 [70pts] Examples of finding inverse or implicit mappings. I hope these help illustrate the inverse and implicit mapping theorems which we will study later in the course. The tools you need to solve these are precalculus and an understanding of the definitions in Chapter 3.

1. suppose $f(x) = (x-2)^2$ for all $x \in \mathbb{R}$. Find a local inverse for f relative to some subset of $dom(f)$. Graph the local inverse and the function.
2. Suppose $F : \mathbb{R}^3 \to \mathbb{R}^2$ is defined by $F(x,y,z) = (e^x, zy^2)$ for all $(x,y,z) \in \mathbb{R}^3$. Find a local inverse for F the given mapping. State the domain U for which $F|_U$ is invertible.
3. Given that $x^2 + y^2 + z^2 = 1$ find $y = f(x,z)$ for some function f. State the $dom(f)$.
4. Given that $x^2 + y + z = 1$ find $x = f(y,z)$ for some function f. State the $dom(f)$.
5. Suppose $F(x,y,z) = (x^2 + y^2, y + z)$. If $F(x,y,z) = (1,4)$ then find $x = g(z)$ and state $dom(g)$.
6. Suppose $F(x,y,z) = (x^2 + y^2, y + z)$. If $F(x,y,z) = (1,4)$ then find $x = g(y)$ and state $dom(g)$.

Comment: if the last problem doesn't make sense let me know.

Problem A8 [35pts] It is at times useful to realize:

1. dot-product is a continuous mapping
2. cross-product is a continuous mapping.

Prove these assertions by utilizing the theorems in my notes. In other words, you can argue in the style of examples 3.2.20, 3.2.21, 3.2.22. You may find my ϵ_{ijk} notation useful for this problem. (if you want to try an epsilon-delta argument feel free, but I don't recommend it upon further thought)

Problem A9 [45pts] Hint, choose $\delta = 1$ for the first part.

1. Give an epsilon-delta argument to prove contant mappings are continuous.
2. Give an epsilon-delta argument to prove $f(x) = \sqrt{x}$ is continuous for $x > 0$
3. Given $g(x)$ is continuous at x and $g(x) > 0$ prove $h(x) = \sqrt{g(x)}$ is continuous.

For part (3.) you could just prove that $f(x) = \sqrt{x}$ is continuous on $[0, \infty)$ then use the theorem from Edwards about the composite of continuous functions.

Problem A10 [50pts] Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous at $a \in \mathbb{R}$ and $f(a) \neq 0$.

1. Give an epsilon-delta arguments that $h = 1/f$ is continuous at a.
2. Give an epsilon-delta argument that $g = f^2$ is continuous at a.
3. Use a result from the notes about the product function to prove that $g = f^2$ is continuous at a.

Problem B1 [100pts] orthogonal transformations. A **line $\mathcal{L}$ with direction** v is defined to be the set $\mathcal{L} = \{tv + b \mid t \in \mathbb{R},\}$ where $v, b \in \mathbb{R}^n$ and $v \neq 0$. A **line segment $\mathcal{L}_{pq}$ from** $p \in \mathbb{R}^n$ **to** $q \in \mathbb{R}^n$ is defined to be the set $\mathcal{L}_{pq} = \{p + t(q - p) \mid t \in [0, 1],\}$.

1. show that if $L : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation such that $L(v) \neq 0$ then $L(\mathcal{L})$ is a line in $\mathbb{R}^m$.
2. show that if $L : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation such that $L(p - q) \neq 0$ then $L(\mathcal{L}_{pq})$ is a line segment from $L(p)$ to $L(q)$ in $\mathbb{R}^m$.
3. Suppose $L : \mathbb{R}^3 \to \mathbb{R}^3$ is an injective linear transformation.**Show that if $P \subseteq \mathbb{R}^3$ is a paralellogram then $L(P)$ is a paralellogram and**

$$Area(L(P)) = |det(A)|Area(P).$$

Hint: the area of a paralellogram is given by $|v \times w|$ where v, w are vectors corresponding to the sides of P. Moreover, $det(A) = \sum_{ij} \epsilon_{ij} A_{1i} A_{2j}$. Two properties of the cross product are $x \times (v + w) = x \times v + x \times w$ and $x \times (cv) = c(x \times v)$. Repeated application of those properties justifies the calculation below:

$$x \times Aw = x \times (\sum_{ij} A_{ij} w_j e_i) = \sum_{ij} A_{ij} w_j (x \times e_i)$$

Moreover, $e_i \times e_j = \sum_k \epsilon_{ijk} e_k$. All of this said, this is a 2×2 problem, if you are not comfortable with the ϵ-notation you could just work it out algebraically in terms of an arbitrary matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

4. The volume of n-dimensional paralellogram can be defined in terms of the determinant. If the n-paralellogram is placed with a corner at the origin such that it has vectors $v_1, v_2, \ldots, v_n$ pointing along the sides then $vol(P) = |det[v_1|v_2|\cdots|v_n]|$. **Show that if $L : \mathbb{R}^n \to \mathbb{R}^n$ is an injective linear transformation with standard matrix A then $vol(L(P)) = |det(A)|vol(P)$.**

Notice that in the case of a box B with sides le_1, we_2, he_3 we can easily recover the handy-dandy formula $vol(B) = det(le_1|we_2|he_3) = lwhdet(e_1|e_2|e_3) = lwhdet(I) = lwh$.

Problem B2 [100pts] Prove the closed case of Theorem 3.3.1, Prove that $\overline{B_1(0)} \subset \mathbb{R}^n$ is homeomorphic to $\mathbb{R}^n$

Problem C1 [100pts] animate paths. (unfinished)

Problem C2 [100pts] draw parametric surfaces, level curves vs. graphs. (unfinished)