Name:

MATH 121-002, FEB. 24, 2010, TEST I

Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Thanks and enjoy.

Problem 1 [75pts] Multiply and leave polynomials in standard form.

(a.) $f_1(x) = (x+2)(x-9)$

$= x^2 + 2x - 9x - 18$

$= \boxed{x^2 - 7x - 18}$

(b.) $f_2(x) = (2x+3)(x^2+4)$

$= \boxed{2x^3 + 3x^2 + 8x + 12}$

(c.) $f_3(x) = 2(x+1)(x+2) + x(x^2+4x+5)$

$= 2(x^2+3x+2) + x^3 + 4x^2 + 5x$

$= \boxed{x^3 + 6x^2 + 11x + 4}$

Problem 2 [50pts] Use the laws of exponents to find the valuesof A and B given that

$$x^A y^B = \frac{xy^3}{\sqrt{y^5 x}} = x^1 y^3 y^{-\frac{5}{2}} x^{-\frac{1}{2}}$$

$= x^{1-\frac{1}{2}} y^{3-\frac{5}{2}}$

$= x^{\frac{1}{2}} y^{\frac{1}{2}} \quad \therefore \boxed{A = B = \frac{1}{2}}$

Problem 3 [100pts] Find the vertex of the parabola $\underbrace{y = (x+2)^2 - 3}$.

$\longrightarrow \boxed{(-2, -3)}$

Problem 4 [200pts] Factor the polynomials below as much as is possible over $\mathbb{R}$.

(a.) $f_1(x) = x^2 + 4x + 3$

$= (x+1)(x+3)$.

(b.) $f_2(x) = x^3 + 5x^2 + 6x$

$= x(x^2+5x+6) = x(x+3)(x+2)$.

(c.) $f_3(x) = x^4 - 9 = (x^2-3)(x^2+3)$

$= (x-\sqrt{3})(x+\sqrt{3})(x^2+3)$.

(d.) $f_4(x) = (x+3)x^2 - (x+3)$

$= (x+3)(x^2-1)$

$= (x+3)(x+1)(x-1)$.

Problem 5 [60pts] Find all real solutions to the equations below:

(a.) $0 = x^2 + 4x + 3 = (x+1)(x+3) = 0$

$\therefore x = -1,\ x = -3$.

(b.) $0 = x^3 + 5x^2 + 6x = x(x+2)(x+3)$

$\therefore x = 0, -2, -3$.

(c.) $0 = x^4 - 9 = (x-\sqrt{3})(x+\sqrt{3})(x^2+3)$

$\therefore x = \sqrt{3}, -\sqrt{3}$.

(d.) $0 = (x+3)x^2 - (x+3)$

$0 = (x+3)(x+1)(x-1) \Rightarrow x = -3, -1, 1$.

$y = (x+2)^2 - 3$

$y = x^2+4x+1 = 0$

$x = \dfrac{-4 \pm \sqrt{16-4}}{2}$

$x = -2 \pm \sqrt{3}$

$\boxed{(-2+\sqrt{3}, 0)\ \&\ (-2-\sqrt{3}, 0)}$

Problem 6 [40pts] find the points at which the graphs of the functions given in ~~Problem 3~~ Problem 4 touch the x-axis.

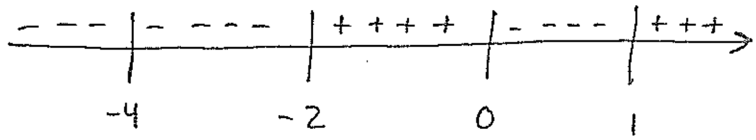
$y = f_1(x)$ | $(-1, 0)$ and $(-3, 0)$ are the zeros (a.k.a. x-intercepts)

$y = f_2(x)$ | $(0,0), (-2,0), (-3,0)$ are the x-intercepts.

$y = f_3(x)$ | $(0, \sqrt{3}), (0, -\sqrt{3})$ are the intercepts of x-axis & graph.

$y = f_4(x)$ | $(-3, 0), (-1, 0), (1, 0)$ ← places $y = f_4(x)$ crosses x-axis.

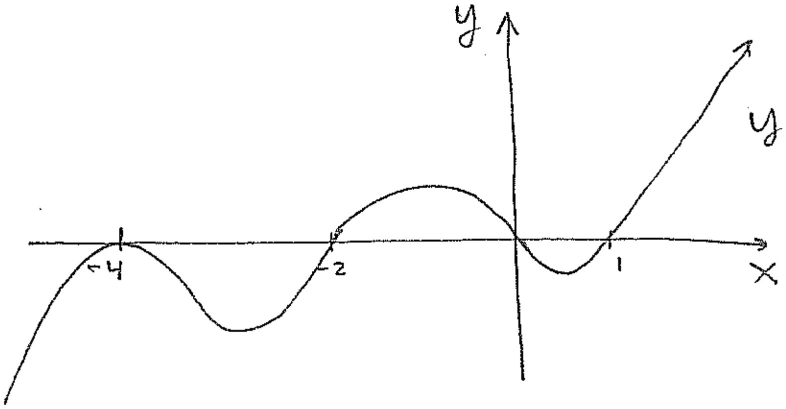
Problem 7 [200pts] Let $f(x) = (x+4)^4(x+2)^3x(x-1)$. Draw the sign chart and then use the sign chart to help sketch the graph of $f(x)$.



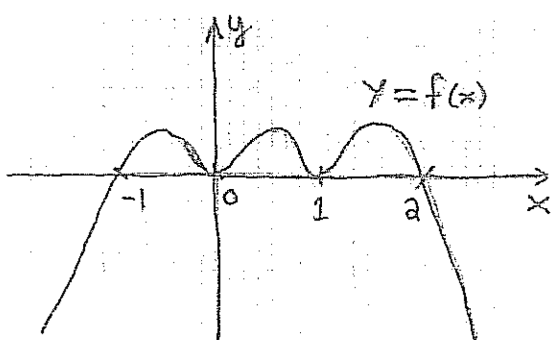
$f(x) = (x+4)^4(x+2)^3x(x-1)$

$f(-5) = (-1)^4(-3)^3(-5)(-6) < 0$

$f(2) \geq 0.$



Problem 8 [75pts] Given the graph pictured below, circle the formula which could give such a graph.



generally for $n = 1, 2, \ldots$

$$\boxed{\begin{array}{ll} \text{need} & (x)^{2n} \\ \text{need} & (x-1)^{2n} \\ \text{need} & (x+1)^{2n+1} \\ \text{need} & (x-2)^{2n+1} \end{array}}$$

both seem to work

need to check sign somewhere.

(a.) $f(x) = -3(x+1)x^2(x-1)^4(x-2)^3 = -3(x+1)x^2(x-1)^4(x-2)^3$

(b.) $f(x) = 3(x+1)x^2(x-1)^4(x-2)^3 = 3(x+1)x^2(x-1)^4(x-2)^3$

(c.) $f(x) = 3(x+1)^2x(x-1)^3(x-2)^2 = 3(x+1)^2x(x-1)^3(x-2)^2$

(d.) $f(x) = (x+1) + x^2 + (x-1)^4 + (x-2)^3 = \underbrace{(x+1) + x^2 + (x-1)^4 + (x-2)^3}$

want $f(-2) < 0$

but for (b.) $f(-2) > 0$

$\therefore$ it must be (a.).

I rewrote them to make the exponents clearer.

Problem 9 [100pts] Let $f(x) = x^3 + 6x^2 - x - 6$. Note that $f(1) = 0$. Factor $f(x)$ completely over $\mathbb{R}$.

$$= x^2(x+6) - (x+6)$$
$$= (x^2-1)(x+6)$$
$$= \underline{(x-1)(x+1)(x+6)}.$$

Problem 10 [100pts] Let $g(x) = x^4 + 3x^3 + 8x^2 + 12x + 16$. Note that $g(2i) = 0$ where $i^2 = -1$. Factor $g(x)$ by using a theorem about complex roots of real polynomials.

Note $g(2i) = 0 \Rightarrow g(-2i) = 0 \Rightarrow (x+2i)(x-2i) = x^2+4$ factors g

$$
\begin{array}{r|l}
 & x^2 + 3x + 4 \\
\hline
x^2+4 & x^4 + 3x^3 + 8x^2 + 12x + 16 \\
 & x^4 \qquad\quad + 4x^2 \\
\hline
 & 3x^3 + 4x^2 + 12x + 16 \\
 & 3x^3 \qquad\quad + 12x \\
\hline
 & 4x^2 \qquad\quad + 16 \\
 & 4x^2 \qquad\quad + 16 \\
\hline
 & 0
\end{array}
$$

$$\therefore \quad \boxed{g(x) = (x^2+4)(x^2+3x+4)}$$

$\left(\begin{array}{l} x^2+3x+4 \text{ is irreducible since} \\ x^2+3x+4 = 0 \text{ has } \underline{\text{complex}} \text{ sol}^{\text{n}}\text{'s.} \end{array} \right)$

Problem 11 [100pts] Write down an example of a polynomial function as instructed below:

(a.) polynomial function $p(x)$ with zeros at $x = 3, -2, 0$.

$$\underline{p(x) = (x-3)(x+2)x}.$$

(many answers possible)

(b.) polynomial function $q(x)$ with a complex zero of $x = 1 + 3i$ and a real zero of $x = 2$ such that the graph "bounces" at $(2, 0)$

$$\underline{q(x) = (x^2 - 2x + 10)(x-2)^2}.$$

must come with zero of $1-3i$ as well

$$(x-1-3i)(x-1+3i) =$$
$$= x^2 - x + \cancel{3ix} - x + 1 - \cancel{3i} - \cancel{3ix} + \cancel{3i} - 9i^2$$
$$= x^2 - 2x + 10$$
$$= (x-1)^2 + 9 \quad \text{(just checking)}$$

Problem 12 [100pts] Find the equation of a line which passes through the points $(2, 3)$ and $(6, 7)$.

$$y = mx + b$$
$$3 = 2m + b$$
$$7 = 6m + b$$

$\longrightarrow 4 = 4m \quad \therefore \underline{m = 1}.$

$\hookrightarrow b = 7 - 6m = 7 - 6 = \underline{1 = b}.$

$$\therefore \boxed{y = x + 1}$$

Problem 13 [75pts] Solve the inequality $2x - 3 \geq -x + 2$.

$$\Rightarrow 2x - 3 + x - 2 \geq -x + 2 + x - 2$$
$$\Rightarrow 3x - 5 \geq 0$$
$$\Rightarrow 3x \geq 5$$
$$\Rightarrow \boxed{x \geq 5/3}$$

Remark: I don't know why I did this, could just go to directly.

Problem 14 [75pts] Solve the inequality $\underbrace{x^2 + 4x + 3}_{} \leq 0$.

$$(x+3)(x+1) \leq 0$$

$$+++ \;\big|\; - - - - \;\big|\; + + + \quad x^2+4x+3$$

$$\quad -3 \qquad\qquad -1$$

$\therefore$ $\boxed{-3 \leq x \leq -1 \quad \text{gives} \quad x^2+4x+3 \leq 0}$

or $x \in [-3, -1]$ if you prefer.

Problem 15 [75pts] Find the domain of $f(x) = \sqrt{\underbrace{x^2 + 4x + 3}_{}}$.

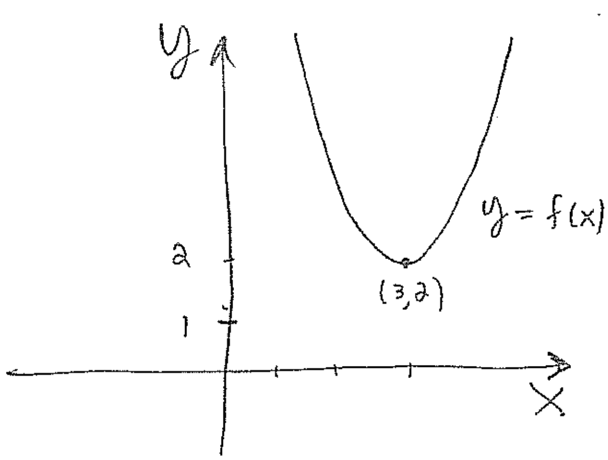
need $x^2+4x+3 \geq 0$

look at sign chart from 14.

either $x \leq -3$ or $x \geq -1$

$\therefore$ $\underline{\text{dom}(f) = (-\infty, -3] \cup [-1, \infty)}$

Problem 16 [75pts] Find the domain and range of $f(x) = x^2 - 6x + 11$.



$= (x-3)^2 + 2$

$\therefore$ vertex at $(3,2)$

and $A = 1 > 0$ so

it opens upward.

$\boxed{\begin{array}{l} \text{range}(f) = [2, \infty) \\ \text{dom}(f) = \mathbb{R} = (-\infty, \infty) \end{array}}$

($f(x)$ makes sense for all $x \in \mathbb{R}$.)