

Name:

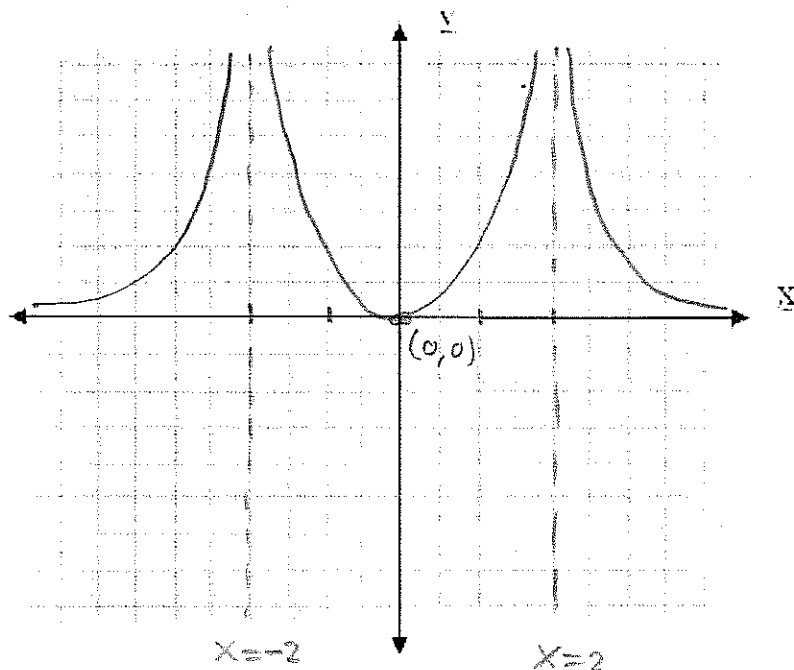
MARCH 29, 2010

TEST II

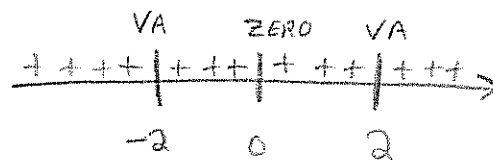
MATH 121-002, FEB. 24, 2010, TEST I

Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Thanks and enjoy. Remember, no graphing calculators permitted.

**Problem 2** [200pts] Simplify the rational function  $f(x) = \frac{x^2}{(x^2-4)^2}$  and construct the sign-chart. Use the sign-chart to help graph  $y = f(x)$ . Make sure to label all the vertical asymptotes and zeros.

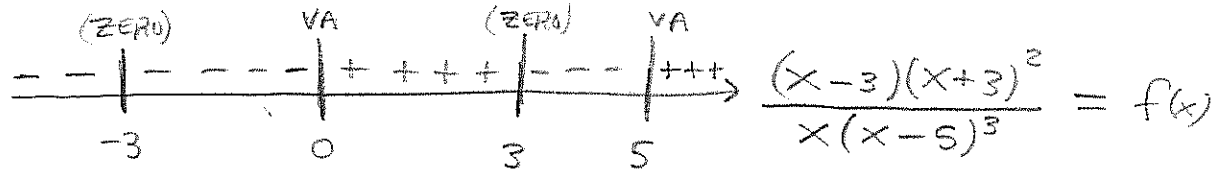


$$f(x) = \frac{x^2}{[(x-2)(x+2)]^2}$$
$$= \frac{x^2}{(x-2)^2(x+2)^2}$$



**Problem 3** [100pts] Find all real  $x$  that solve the inequality below: (using a sign-chart to aid your logic is encouraged!)

$$\frac{(x-3)(x+3)^2}{x(x-5)^3} \leq 0$$



note  $f(-6) = \frac{(-)(-)^2}{(-)(-)^3} < 0$

$$\therefore \boxed{x < -3 \text{ or } 3 \leq x < 5}$$

a.k.a.  $\boxed{(-\infty, -3) \cup [3, 5)}$

**Problem 4** [100pts] Find numbers for  $A, B$  and  $C$  that make the equality below true.

$$4 \ln(x^3) + \frac{1}{3} \ln(x+6) - 2 \ln(x^3-9) = \ln \left[ \frac{x^A(x+6)^B}{(x^3-9)^C} \right]$$

$$\ln(x^{12}) + \ln \sqrt[3]{x+6} + \ln \frac{1}{(x^3-9)^2} = \ln \left( \frac{x^{12}(x+6)^{1/3}}{(x^3-9)^2} \right)$$

$$\therefore \boxed{A=12, B=1/3, C=2}$$

**Problem 6** [150pts] Solve  $2^{3x-15} = 8$ .

$$8 = 2^3 = 2^{3x-15}$$

$$\text{1-1 prop.} \Rightarrow 3 = 3x - 15$$

$$\Rightarrow 1 = x - 5$$

$$\Rightarrow \boxed{x = 6}$$

Problem 7 [150pts] Solve  $3^{4x+3x^2} = \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$

$$3^{4x+3x^2} = 3^{-3}$$

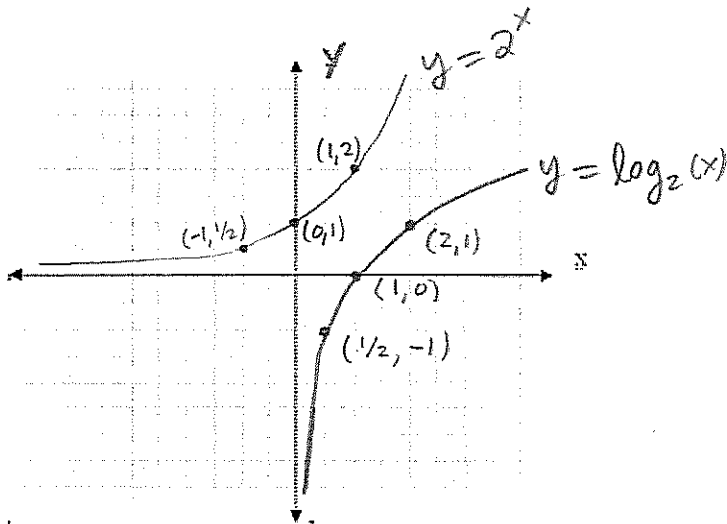
$$4x + 3x^2 = -3$$

$$x^2 + 4x + 3 = 0$$

$$(x+3)(x+1) = 0$$

$$\Rightarrow \boxed{x = -3 \text{ or } x = -1}$$

Problem 8 [100pts] Graph  $y = 2^x$  and  $y = \log_2(x)$ . Include at least 3 specific points on both graphs.



Problem 9 [100pts] Solve  $\log_5(x) = 2$ .

$$5^{\log_5(x)} = 5^2$$

$$\boxed{x = 25}$$

Problem 10 [100pts] Solve  $\log(x+9) + \log(x) = 1$ .

$$\log(x^2 + 9x) = 1$$

$$x^2 + 9x = 10^1 = 10$$

$$x^2 + 9x - 10 = 0$$

$$(x+10)(x-1) = 0 \quad \therefore \underline{x = -10} \text{ or } \underline{x = 1}$$

Notice that  $x = -10$  is not a sol<sup>n</sup> since  $\log(-10)$  d.n.e as a real # hence  $\boxed{x = 1}$  is the only sol<sup>n</sup>.

Problem 11 [200pts] Find the inverse function of  $f(x) = 3 + \frac{x}{x-3}$ .

$$y = 3 + \frac{x}{x-3}$$

$$x = 3 + \frac{y}{y-3}$$

$$(x-3)(y-3) = y$$

$$xy - 3y - 3x + 9 = y$$

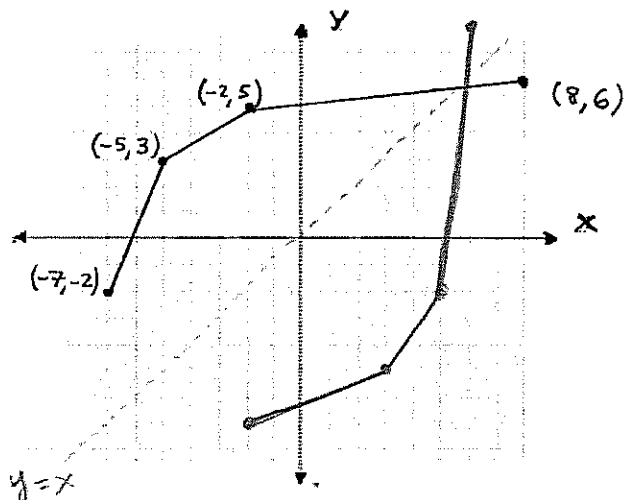
$$4y - xy = 9 - 3x$$

$$y(4-x) = 9-3x$$

$$y = \frac{9-3x}{4-x}$$

$$\therefore \boxed{f^{-1}(x) = \frac{9-3x}{4-x}}$$

Problem 13 [100pts] Given the graph  $y = f(x)$  below, construct the graph of  $y = f^{-1}(x)$ .



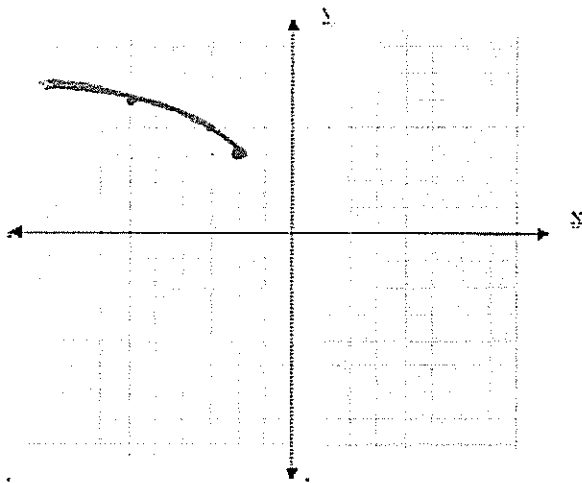
Problem 14 [100pts] Let  $f(x) = x^2 + \sqrt{x}$  and  $g(x) = \frac{1}{x+1}$ . Find the formulas for

$$1. (f-g)(x) = f(x) - g(x) = \boxed{x^2 + \sqrt{x} - \frac{1}{x+1}}$$

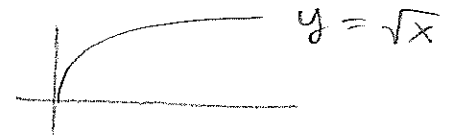
$$2. (fg)(x) = f(x)g(x) = (x^2 + \sqrt{x}) \frac{1}{x+1} = \boxed{\frac{x^2 + \sqrt{x}}{x+1}}$$

$$3. (f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x+1}\right) \\ = \boxed{\left(\frac{1}{x+1}\right)^2 + \sqrt{\frac{1}{x+1}}}$$

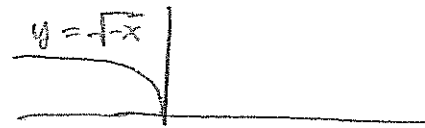
**Problem 15** [100pts] Take the graph  $y = \sqrt{x}$  and reflect it across the  $y$  axis. Secondly, shift the graph up by 3 units. Third, shift the graph 2 units left. Graph the result of this three-step transformation and give the formula for this function.



$$y = 3 + \sqrt{-x-2}$$



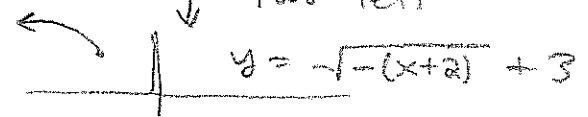
↓ reflect across  $y$ -axis



↓ up 3



↓ two left



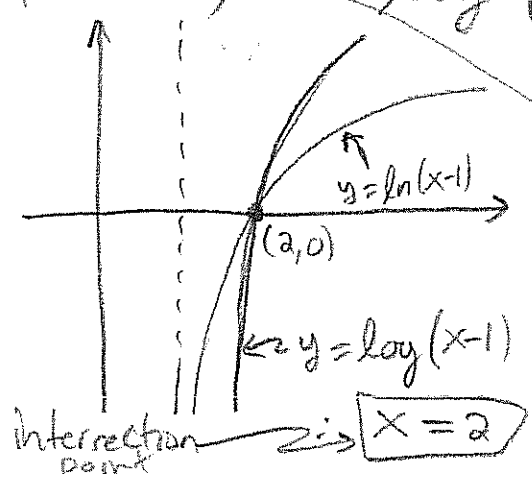
**Bonus** [50pts] Solve  $\sqrt{x+6} = \sqrt{6-x}$ .

$$\begin{aligned} x+6 &= 6-x \\ 2x &= 0 \\ \therefore x &= 0 \end{aligned}$$

**Bonus** [50pts] Find all real solutions of  $\log(x^2 - 1) - \log(x + 1) = \ln(x - 1)$ .

$$\log(x+1)(x-1) - \log(x+1) = \ln(x-1)$$

(graphical soln)



$$\log\left(\frac{(x+1)(x-1)}{x+1}\right) = \ln(x-1)$$

$$\log(x-1) = \ln(x-1)$$

$$= \frac{\log(x-1)}{\ln(10)} \quad (\text{algebra soln})$$

$$\Rightarrow \left[1 - \frac{1}{\ln(10)}\right] \log(x-1) = 0$$

$$\therefore \log(x-1) = 0 \Rightarrow x-1 = 1 \Rightarrow x = 2$$