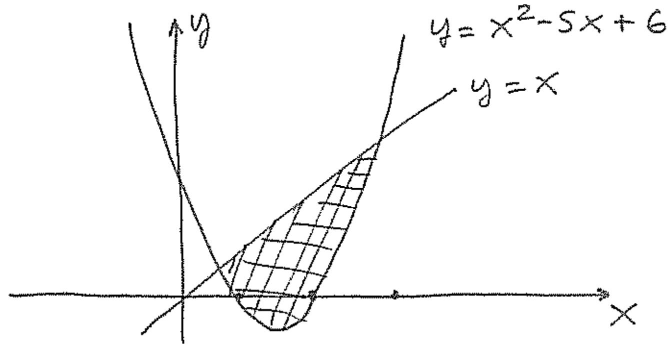
Math 121, Test 3, Section # : Name: .

No graphing calculators or electronic communication of any kind. If you need extra paper please ask. Credit will be awarded for correct content and clarity of presentation. This test has 100 points. Try to at least attempt each part. Please clearly **box** your answer in each question.

1. [100pts] Graph the inequalities $y \geq \underbrace{x^2 - 5x + 6}$ and $y - x \leq 0$. Shade the solution set.

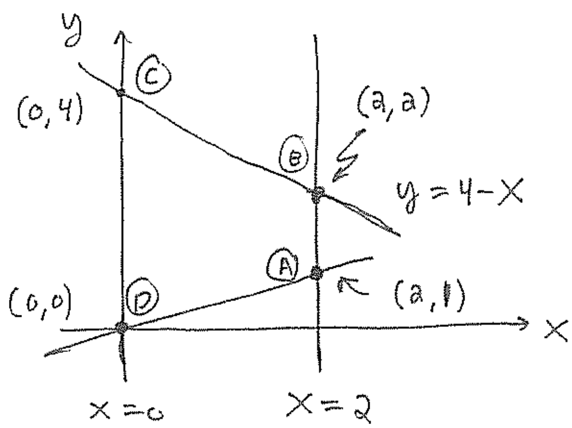
$y \geq (x-2)(x-3) \longleftrightarrow (x-2)(x-3) \leq y$



$x \geq y$

$y \leq x$

2. [200pts] Suppose that $0 \leq x \leq 2$ and $x \leq 2y$ and $y \leq 4 - x$ constrain the objective function $z = x - y + 3$. Find the maximum and minimum values for z subject to the given constraints. Use the method of linear programming to find the solution.



$y \leq 4 - x$ (below this $y = 4 - x$)

$x \leq 2y$

$y \geq \frac{x}{2}$ above $y = \frac{x}{2}$

$x = 2 \Rightarrow y = \frac{2}{2} = 1$

$\therefore$ A $= (2,1)$

$x = 2 \Rightarrow y = 4 - 2 = 2$

$\therefore$ B $= (2,2)$

Linear Programming says max/min at corners,

	x	y	$Z = x - y + 3$
A	2	1	$Z = 2 - 1 + 3 = 4.$
B	2	2	$Z = 2 - 2 + 3 = 3$
C	0	4	$Z = 0 - 4 + 3 = -1$
D	0	0	$Z = 0 - 0 + 3 = 3$

$\therefore$ $\boxed{\begin{array}{l} Z_{max} = 4 \text{ at } (2,1) \\ Z_{min} = -1 \text{ at } (0,4). \end{array}}$

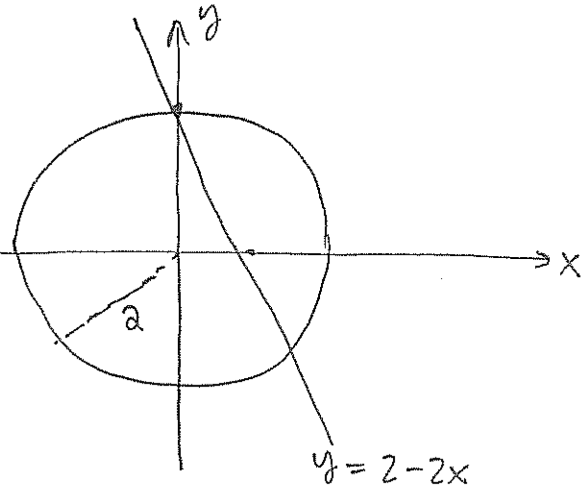
3. [100pts] Find the points of intersection of the graph of the equation $x^2 + y^2 = 4$ and $\underbrace{y + 2x = 2}$. Verify your algebraic solution with a graph of both equations.

$\underbrace{x^2+y^2=4}$

circle

radius

2.



$$y = 2 - 2x$$

$$x^2 + (2-2x)^2 = 4$$

$$x^2 + \cancel{4} - 8x + 4x^2 = \cancel{4}$$

$$5x^2 - 8x = 0$$

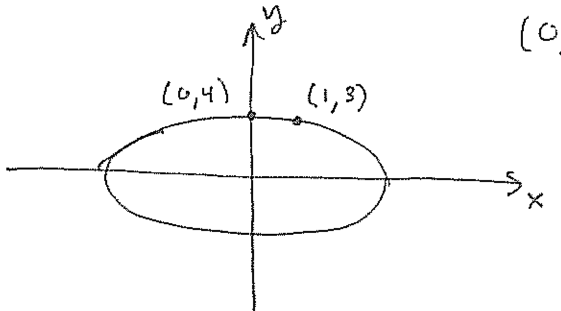
$$x(5x-8) = 0$$

$$\therefore\ x = 0 \quad \text{or} \quad x = 8/5$$

$$\boxed{(0,2) \quad \text{or} \quad (8/5, -6/5)}$$

note, $2 - \frac{16}{5} = \frac{-6}{5}$

4. [100pts] Find an ellipse of the form $x^2/a^2 + y^2/b^2 = 1$ whose graph includes the points (1,3) and (0,4).



$$\boxed{\frac{x^2}{16/7} + \frac{y^2}{16} = 1}$$

$(0,4)$: $0 + \frac{16}{b^2} = 1 \quad \therefore\ 16 = b^2 \ \therefore\ \underline{b = 4}$.

(can choose 4)

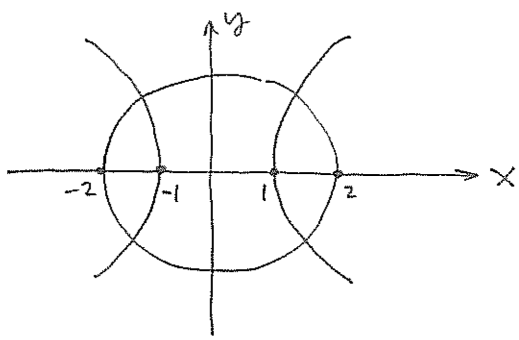
(could also use −4)

$(1,3)$: $\frac{1}{a^2} + \frac{9}{b^2} = 1$

$$\frac{1}{a^2} = 1 - \frac{9}{16} = \frac{16-9}{16} = \frac{7}{16}$$

$$\therefore\ a = \sqrt{\frac{16}{7}} \quad \text{or} \quad a^2 = \frac{16}{7}$$

5. [100pts] Find the points of intersection of $x^2 + y^2 = 4$ and $x^2 - y^2 = 1$. Draw a picture and think about your algebra. Are you sure you got all the solutions?



$$\left.\begin{array}{l} x^2 + y^2 = 4 \\ x^2 - y^2 = 1 \end{array}\right\} \rightarrow 2x^2 = 5$$

$$x^2 = 5/2$$

$$\underline{x = \pm\sqrt{\frac{5}{2}}}.$$

$$y^2 = x^2 - 1 \Rightarrow \underbrace{\underline{y = \pm\left(\frac{5}{2} - 1\right)}}.$$

$$\underline{y = \pm\frac{3}{2}}.$$

Sol'ns : $\boxed{\left(\sqrt{\frac{5}{2}}, \frac{3}{2}\right), \left(\sqrt{\frac{5}{2}}, \frac{-3}{2}\right), \left(-\sqrt{\frac{5}{2}}, \frac{3}{2}\right), \left(-\sqrt{\frac{5}{2}}, \frac{-3}{2}\right)}$

6. [100pts] Find the equation of a parabola through the points (0,1), (1,3), (-1, 4).

$$y = Ax^2 + Bx + C$$

$(0,1)$: $1 = A(0) + B(0) + C$ $\therefore$ $\underline{C = 1}$.

$(1,3)$: $3 = A + B + 1$

$(-1,4)$: $4 = A - B + 1$

$\xrightarrow{\text{add}}$ $7 = 2A + 2 \rightarrow \underbrace{5 = 2A}$

$\underline{A = 5/2}$.

Also, $B = 2 - A = 2 - \frac{5}{2} = \underline{\frac{-1}{2} = B}$

$\therefore \boxed{y = \frac{5}{2}x^2 - \frac{1}{2}x + 1}$

7.[150pts]Solve the following system of equations:

$x + 2y = 5$

$\underbrace{-x + 4y = 7}$

$\xrightarrow{\text{add}}$ $6y = 12$ $\therefore$ $y = \frac{12}{6} = 2$.

$x = 4y - 7 = 8 - 7 = 1$ $\therefore$ $\boxed{x = 1, \quad y = 2}$

8.[150pts]Solve the following system of equations:

$x + 2y + z = 4$

$-3x + y - z = 0$

$\underbrace{x - y - z = 0}$

$\xrightarrow{\text{add}}$ $-2x + 3y = 4$ $\therefore$ $3y = 4 + 2x \Rightarrow y = \frac{4+2x}{3}$ (I)

$\xrightarrow{\text{add}}$ $-2x - 2z = 0$ $\therefore$ $\underline{-x = z}$. $\therefore$ $\underline{z = -x}$. (II)

substitute (I) & (II) into the third eq^n,

$$x - \frac{4+2x}{3} + x = 0 \Rightarrow 2x - \left(\frac{4+2x}{3}\right) = 0$$

$6x - 4 - 2x = 0$

$4x = 4$

$\therefore$ $\underline{x = 1}$.

$\Rightarrow$ $\underline{z = -1}$

$\Rightarrow$ $\underline{y = 2}$.

$\boxed{x = 1, \quad y = 2, \quad z = -1}$

9.[100pts]Given the information below, read off the solutions.

$$\begin{aligned} 3x - y + z &= 0 \\ x + 2y + z &= 4 \\ 2x - 2y - 2z &= 0 \end{aligned} \quad \text{has} \quad rref\begin{pmatrix} 3 & -1 & 1 & 0 \\ 1 & 2 & 1 & 4 \\ 2 & -2 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{pmatrix} \longleftrightarrow \boxed{\begin{aligned} x &= 1 \\ y &= 2 \\ z &= -1 \end{aligned}}$$

10.[100pts] Suppose that $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ has the multiplicative inverse $A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$. Solve the following system of equations given in matrix form below by multiplying by the inverse matrix given above.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{bmatrix} 3-2+1 \\ 2 \\ -1 \end{bmatrix} = \underline{\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}}.$$

$$\boxed{x = 2, \quad y = 2, \quad z = -1}$$

11. [200pts] Suppose that $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 8 \\ 0 & 2 \end{pmatrix}$. Calculate

a.) $AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 8 \\ 0 & 2 \end{pmatrix} = \underline{\begin{bmatrix} 5 & 12 \\ 15 & 32 \end{bmatrix}}.$

b.) $A + 2B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \underbrace{2\begin{bmatrix} 5 & 8 \\ 0 & 2 \end{bmatrix}}_{\begin{bmatrix} 10 & 16 \\ 0 & 4 \end{bmatrix}} = \underline{\begin{bmatrix} 11 & 18 \\ 3 & 8 \end{bmatrix}}$

c.) $\det(A) = 4 - 6 = \boxed{-2}$

d.) $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix} = A^{-1}}$

if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then ↑

Math 121, Test 3, Section # : Name: .

12. [100pts] Calculate the following:

$$\det\begin{pmatrix}0 & -1 & 1\\ 0 & 2 & 1\\ 3 & 0 & 1\end{pmatrix} = 0\cdot\det\begin{bmatrix}2 & 1\\ 0 & 1\end{bmatrix} + 1\det\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix} + 1\det\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}$$

$$= 1(-3) + 1(-6)$$

$$= \boxed{-9}$$

BONUS[50pts]: What's the area of a triangle with vertices (0,-1), (0,2) and (3,0)?

$$A = \pm\frac{1}{2}\det\begin{bmatrix}0 & -1 & 1\\ 0 & 2 & 1\\ 3 & 0 & 1\end{bmatrix} = \pm\frac{1}{2}(-9) \Rightarrow \boxed{A = \frac{9}{2}}.$$