REVIEW FOR TEST I OF CALCULUS I:

The first and best line of defense is to complete and understand the homework and lecture examples. Past that my old test might help you get some idea of how my tests typically look. Most of the test will be like problems you've done before, they may not be the same format but they should require the same skill set. This test covers the material that was covered by the homework and lecture examples as well as "precalculus" in general.

Background Knowledge:

1. Memorize all the basic graphs. Many of you have relied on the text for looking up the formulas and graphs; you will not have that luxury on the test. So memorize the graphs and check yourself as you study. Brush-up on transformation of graphs so you can quickly graph simple curves like $y = x^2 + 3$ or $y = \sqrt{x-1}$.

2. By "basic graphs" I mean that I expect you know all the important features such as domain, range and vertical asymptotes for the following list of elementary functions
   - basic polynomials like $p(x) = 1, x, x^2, x^3, x^4, \ldots$
   - basic rational functions like $r(x) = 1/x, 1/x^2, 1/(x-2), 1/(x+3)^2, \ldots$
   - root functions such as $f(x) = \sqrt{x}$
   - absolute value functions
   - exponential functions
   -hyperbolic sine and cosine
   - logarithmic functions
   - sine, cosine and tangent
   - inverse sine, inverse cosine, inverse tangent

   All of the examples above you should be able to graph without much thinking. I also expect you can, with some additional thinking, graph transformations of the graphs given above such as $f(x) = \sin(\pi x), e^x + 3, \ln(x+2), 2|x+6|, \ldots$

3. Know difference between inverse trigonometric functions and reciprocal trigonometric functions. For example, $f(x) = \sin^{-1}(x)$ and $g(x) = \csc(x) = 1/sin(x)$ are entirely different functions. Please end your confusion over this matter before the test.

4. Be able to define the hyperbolic sine and cosine. I don't expect you remember everything about them, but the definition you ought to know for future reference. I might ask you to prove an identity about them as we worked in lecture.

5. Know your basic algebra facts about exponentials and logarithms. Be able to solve transcendental equations like in the homework. Of course I also expect you can factor and do long division if need be. This could come up in the limit of a hard to factor cubic like $1/(x^3+8)$. You should also be able to use the factor theorem to create new examples as in the first homework set.

6. Know how to define new functions from old. In particular, given real-valued functions of a real variable f, g and a constant $c \in \mathbb{R}$ know how to define $cf, f+g, f-g, fg, f \circ g, \frac{f}{g}$ given appropriate data about the functions.

7. Know how to compose functions, including piece-wise defined examples.

Proof-type Responsibilities:

1. Be prepared to state the definition of a function including the domain and codomain.

2. Be prepared to state the definition of the range of a function.

3. Be prepared to state the definition of the graph of a function.

4. Be able to state the definition of an even function. Be able to prove a given function is even.

5. Be able to state the definition of an odd function. Be able to prove a given function is odd.

6. Be prepared to state the definition of a 1-1 or injective function.

7. Know the definition of a strictly increasing function. Be aware of the theorems which connect strictly increasing, 1-1 and invertibility properties of functions. Likewise for strictly decreasing. What does “strictly monotonic” mean?

8. Understand the idea of a local inverse function relative to some subset of the domain of a given function. I assigned two problems in the first homework about this sort of “understanding”.

9. Be prepared to state the definition of the limit at a point in terms of "epsilons and deltas"

10. Be prepared to use the epsilon-delta formal proof technique for examples like we covered in class. It might be quadratic, linear or a square-root type example. I would expect there are two of these: one worth 5% the other 10%. Everybody should get the 5% problem.

11. Be prepared to state the definition of continuity of a function at a point in the interior of its domain. Likewise, be aware of the concept of continuity at boundary points of the domain.

12. Be prepared to state the Intermediate Value Theorem (IVT).

13. Be able to prove a solution *exists* for an equation by use of the IVT.

14. Be prepared to use the Squeeze Theorem.

15. Be prepared to state the definition of the derivative of a function at a point in terms of a limiting process.

16. Be prepared to state the definition of the tangent line through a point, if it exists, in terms of a limiting process.

17. Be prepared to state the definition of the instantaneous velocity at time t_o given the position $s(t)$.

<u>Important Calculations and some advice:</u>

1. Be able to find the equation of tangent line at a point.

2. Definitions of instantaneous velocity and average velocity for one-dimensional motion. Be able to answer questions like those in the homework.

3. Notation! Please make sure to use notation that is clear and correct. I wrote limits when needed and I omitted them once the limit was completed. I expect you to do likewise on the test. The limit notation is shorthand for a precise mathematical operation, you must denote your logic correctly.

4. Since I have not emphasized the "limit laws" I don't expect you to recite them by number on the exam. However, you should be aware of them and able to use them. In other words, you can use them (correctly) as needed on the test. Of course, first you have to follow instructions. If I say to find the tangent line from the definition of derivative and you use the power or chain rule etc… you will not earn points. We'll get to those in due time.

5. Be able to use the full force of your background knowledge to calculate limits which are not indeterminant. Many times we can conclude from a short calculation that the limit blows up to plus or minus infinity. Or, perhaps we can just evaluate the function. Or perhaps the limit point is not even near the domain of the function. These sort of cases do not necessarily need further algebraic investigation. A short sentence will likely suffice to capture the logic of your answer. On the quiz 1 those limits all were found from basic function knowledge, it could be argued there is no work to show. (other people would disagree, however I don't require you to understand the formal definition of an infinite limit for this test, either a graphing or a numerical argument will suffice, or you could use my $x \approx a \pm \delta$ substitution to work out the infinite limits.)

6. Indeterminant limits of the forms $0/0, \infty - \infty, \infty/\infty$ can often be resolved by some algebraic manipulation. I do require you show steps for these sort of limits. However, once the indeterminancy is removed you may simply state the limit. I don't require you to state you are using the "direct substitution property", although it would make me happy to know you knew if you were.

7. For the item above there were about 4 or so main tricks that came up in the homework. Know those tricks and be prepared to use them on the test for similar limits. However, remember not every limit is hard. Sometimes we just evaluate. When do we do that? Whenever we can, of course.

8. I will certainly ask questions about continuity and discontinuity. Make sure you understand as much as possible about continuity. Pay attention to the definition and be sure your arguments about continuity are based on the definition and not some vague geometric or algebraic principle. Before you plug in limit points write limits and explain why you write the equation that you write. I'm thinking of the third homework set in this comment.

9. Perhaps you're finding this advice a little vague. I would advise you to revisit your homework and renew your efforts to understand the basic and moderate problems, then worry about the tricky ones if you have time left.

Good Studying! I would estimate a crude breakdown as follows: 15% epsilon-delta proof, 25% precalculus, 25% indeterminant limits, 25% continuity, 25% tangent lines and derivatives at point, 10% average and/or instantaneous velocity (this is obviously just a crude indication of the importance of various topics, also the epsilon-delta concept may enter other problems even if they are just conceptual rather than a formal "proof"… I doubt more than 25% of the test will directly bear on the epsilon-delta definition of the limit)