REVIEW FOR TEST 2 OF CALCULUS I:

The first and best line of defense is to complete and understand the homework and lecture examples. Past that my old test might help you get some idea of how my tests typically look. Most of the test will be like problems you've done before, they may not be the same format but they should require the same skill set. The page and section numbers on this review refer to my course notes.

Conceptual Fundamentals:

1. Know your limit laws. This is not a huge issue on this test, but it might come up in a problem where the function was case-wise defined. Maybe you need to calculate left and right limits to ensure continuity of a function at some point. Or, perhaps the difference quotient needs to be evaluated by different rules from the left and right, again in such a case we probably need to expand the problems in terms of explicit limits. (see Problem 31)

2. Know the definition of the derivative as a function in terms of a limiting process.

3. What does it mean for a function f to be "k-times continuously differentiable on U"? In other words, know what the notation $f \in C^k(U)$ for $k \in \mathbb{N}$. Also, what does it mean to say that a function f is "smooth on U"? In other words, what does the notation $f \in C^\infty(U)$ imply about the function f? Finally, what do me mean by $f \in C^0(U)$? Note: these qualifiers are not mutually exclusive. In fact, these sets form a *nested sequence of subsets:*
$$C^0(U) \subset C^1(U) \subset C^2(U) \subset \cdots \subset C^\infty(U)$$
This is just language. Find an example for $k = 0, 1, 2, \infty$ as an exercise.

4. Distinguish between the slope of the tangent line and the derivative function.

5. Be able to find the equation of tangent line at a point. Also, what it the normal line through the same point?

6. Be able to use the linearization to approximate functions near some point. Be able to solve problems similar to those given in the Problem Set.

7. Be able to analyze graph of f(x) verses f'(x) like in #3 or 41 of section 3.2.

8. Be able to work related rates word problems like those in the Problem Sets or Examples from lecture.

9. Definition of velocity and acceleration given the position as a function of time.

10. Know your graphs, all the graphical and algebraic items I listed on the first test still can come up here. You need to know the general formula for a quadratic, cubic etc… Graph of sin(x), cos(x), ln(x) and so forth…

11. Notation! Please make sure to use notation that is clear and correct. If you write that a function is equal to its derivative (like cos(x) = -sin(x)) then bad things will happen. Also, you should never write d/dx = stuff in this course. d/dx always acts on something. (I would not be happy if I saw f(x) = cos(x) so d/dx = -sin(x).)

<u>Calculational Foundations:</u>

1. Linearity, linearity, linearity. Be able to split up problems and attack each piece one at a time when the difficulty suggests it is wise. For example, differentiate
$$f(x) = x^{(x\cos(x))} + \sqrt{\tan(x)}(x^2 - x)^3 \sec(x)e^{ax^3}$$
You don't want to do it all at once, split this into two problems then combine the answer at the end; you should explain that is what you are doing with appropriate notation.

2. Memorize all the basic derivatives. Many of you have relied on the text for looking up the formulas; you will not have that luxury on the test. So memorize the formulas to begin and check yourself as you study. (see section 4.12, I expect you memorize all but the last 4 derivatives on that table.)
3. The derivative of the absolute value function is perhaps worth memorizing. This can be derived from the chain-rule on $f(x) = \sqrt{x^2}$, (note this formula may break into many cases if you wish to unravel the details of the quotient $u/|u|$.
$$\frac{d}{dx}|u| = \frac{u}{|u|}\frac{du}{dx}$$

4. Know how to use $\lim_{x\to 0}\frac{\sin(x)}{x} = 1$ to derive similar or related limits via the limit laws. This was Problem 45 (remember, I got stuck and Ginny mocked me)

5. Know the product, quotient and chain rules. Know how to apply them

6. Know difference between explicit and implicit functions of x. Be able to differentiate implicitly and find tangent line to implicit function. For example, be able to find tangent line to $x^2y + xe^y = 7$ at the point (7,0).

7. Be able to use logarithmic differentiation when it is helpful (multiple products or root functions), or indispensable(things like x^{x^x}).

8. Be able to find the tangent line and linearization of a function at a given point (a,f(a)). What is the difference between these items?

9. Make sure you have a graphical intuition about graphs of functions and corresponding derivatives. In particular, what are the ways the derivative may fail to exist at a given point. What about continuity? What role does it play? How are the concepts of differentiability and continuity different?

Proof-type Responsibilities:

1. Be able to prove additivity of the derivative operation; $\frac{d}{dx}[f+g] = \frac{df}{dx} + \frac{dg}{dx}$.

2. Be able to supply a proof of the product rule. This proof begins with the definition of the derivative and I would allow you to assume the result that a differentiable function at x=a is necessarily continuous function at x=a.

3. Given the product rule be able to prove the quotient rule.

4. Given that $\lim_{h\to 0} \frac{\sin(h)}{h} = 1$ and $\lim_{h\to 0} \frac{\cos(h)-1}{h} = 0$ be able to prove that $\frac{d}{dx}(\sin(x)) = \cos(x)$ or $\frac{d}{dx}(\cos(x)) = -\sin(x)$. You may assume the adding angles trigonometric identities for sine and cosine are known.

5. Be able to derive the formulas for the derivatives of the inverse functions as I did in lecture and the notes. In particular, see Section 4.10.

6. Be able to show the derivatives of the reciprocal trig. Functions are what they are on the basis of the quotient rule applied to quotients of sine and cosine. See, for example, Ex. 4.7.2 and 4.7.5.

Finally, consider the Problem Sets, if I thought it was interesting for the Problem Sets then it's likely that those problems are also test-worthy.

What will be on the test for sure:
1. a related rates problem
2. a linearization problem
3. two proof responsibility questions
4. logarithmic differentiation
5. implicit differentiation (maybe with finding tangent or normal line)
6. most of the basic derivatives (this is a lot of the test)
7. a case-wise defined example where explicit left and right limits are necessary.