REVIEW FOR TEST 4 OF CALCULUS I:

The first and best line of defense is to complete and understand the homework and lecture examples. Most of the test will be like problems you've done before, they may not be the same format but they should require the same skill set.

• the test is likely to divide up about 10% initial value problems,10% physics of motion in one-dimension 50% integration, 30% area.

Initial Value Problems:

1. Be able to calculate the solution to $\frac{dy}{dx} = f(x)$ given some initial condition for y. Of course the function f(x) would need to be something we can antidifferentiate. Here the idea is to fit the constant of the indefinite integral by applying the initial condition.

2. Alternatively you can think about this problem in view of definite integrals as we have more recently. See Example 8.1.1 and contrast it with Example 7.1.6 or Example 7.1.8. (Problem 91 gives good pure math presentation of this calculational idea)

3. Example 7.1.10 was fun, but that sort of thinking would only appear in a bonus question.

Acceleration, Velocity, Position:

4. Be able to calculate the velocity and position as a function of time if given the acceleration as a function of time and an initial position and velocity.

5. Be able to calculate the distance and/or displacement relative to a particular time interval given the velocity as a function of time.

Integration:

1. Be able to carefully state the definition of the definite integral as the limit of Riemann sums. (see Definitions 7.2.10, 7.2.18 and 7.2.19, you only need to memorize the first part of the definition, up to but not including "We also define for…")

2. You should also know how the points $x_0, x_1, \ldots, x_n$ are found once the interval of integration $[a, b]$ is given. Be able to calculate left or right end points rules for small n. (make sure to have a calculator for this test, as always, scientific, non-graphing)

3. Section 7.2.1 on sequences and infinite sums was included to hopefully clarify the notation of the Riemann sum, technically much is missing from the complete story of that section and consequently from the very definition of the definite integral. In calculus II we will return to sequences and infinite sums and dig much deeper. (for this test you just need to understand the left, right, midpoint, and Riemann sums as mentioned in other items…) In summary, the only sort of infinite sums I expect you can calculate are those that can be identified as definite integrals, see Problem 92 and 93. Also 94.

4. Antiderivatives or indefinite integration: know your basic antiderivatives. I might ask all of them. See section 7.1.2 for an exhaustive list of the basic antiderivatives. Hyperbolics and inverse hyperbolics are fair game.

5. Know your properties of definite integration. See section 7.2.4. Problem 95 & 96.

6. What is an "area function" (see Defn. 7.3.1), in truth it's a signed area function to be totally truthful but I don't dwell on that in that part of the notes… see 7.4.1 for more on signed-area vs. area.

7. Know FTC I.

8. Know FTC II. Be able to apply it with confidence (7.4).

9. Know how to differentiate integrals with variable bounds, FTC I is simplest case but generally need chain rule. See Theorem 7.3.7 and the examples that follow it.

10. Notice there are absolute value bars in $\int \frac{1}{x} dx = \ln|x| + c$. They matter.

11. Know the properties for definite and indefinite integrals. In particular, recall that we need the last property to deal with piecewise defined functions.

$$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$$
$$\int cf(x)dx = c\int f(x)dx, \ c \text{ constant}$$
$$\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$
$$\int_a^b cf(x)dx = c\int_a^b f(x)dx, \ c \text{ constant}$$
$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx, \ a < b < c \text{ constants}$$

12. U-substitutions. Know how to do problems like the homework and examples from lecture, this will be a significant part of the test. Most of Problem Set 12.

13. Know about the two main methods to calculate definite integrals involving u-substitution. I may ask a question which forces you to change the bounds. I explain at the start of my section 7.5.2, I have primarily used approach 2 this semester. One exception would be my solution to Problem 100c.

<u>Area Bounded by Curves:</u>

1. First and foremost be able to graph curves similar to those encountered in lecture and/or homework. Be prepared to show your work. Find intersection points using algebra where appropriate.

2. You may be called upon to graph one of those "basic" functions from precalculus such as the exponential, sine, cosine, natural logarithm, and so forth. Ignorance of the graphs of these functions could be a major stumbling block in setting up a problem.

3. Be prepared to draw a picture which illustrates the typical rectangle, know how to find the formula for the infinitesimal area from the picture. Horizontal or vertical strips, when to use functions of x or functions of y, how do you choose?

4. Where do the bounds for integrating dA come from? Hint: this is why I require you draw the picture each and every time.

Volumes: (not on test 4, but this is for the final exam)

1. First and foremost be visualize to volumes similar to those encountered in lecture and/or homework. Be prepared to show your work. Find intersection points using algebra where appropriate.

2. For shapes like cones and pyramids know how to use linearity to find the needed formula for dA.

3. Can you find the volume of a sphere?

4. Be able to find the volume of solids of revolution. To begin with this requires we have a good picture of the area we wish to rotate. Then it is important to understand how the axis of rotation relates to the picture. There are many possibilities, again when do we need to work with functions of x and when functions of y? The lecture examples and homework contain more than enough variety to prepare for the test.