Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Thanks. You are allowed the use of a scientific (non-graphing) calculator. No electronic communication devices of any kind permitted, no IPODs, Zunes, Walkmans etc... This is a timed test and time is likely to be an issue for you, budget your time wisely. There are 150pts to earn on this exam.

Problem 1 [6pts] Simplify the expressions below into the format cx^n where c, n are constants. 1:21

$(a.)\ \dfrac{1}{3\sqrt[5]{x^2}} = \underline{\tfrac{1}{3}x^{\frac{-2}{5}}}$, $(b.)\ \dfrac{2}{7x^3} = \underline{\tfrac{2}{7}x^{-3}}$, $(c.)\ \sqrt[3]{5x^8} = \underline{\sqrt[3]{5}\,x^{8/3}}$.

Problem 2 [3pts] Suppose that f is a one-one function and $a, b \in dom(f)$. If $f(a) = f(b)$ then is it true that $a = b$? (one sentence should do.)

YES, this was our def$^{\underline{n}}$ of a 1-1 fnct.

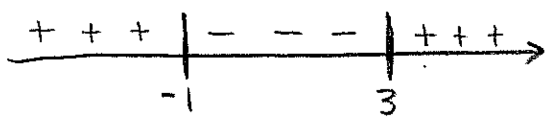
Problem 3 [7pts] Suppose that $f(x) = x^2$ and $dom(f) = [-1, 1]$ is this function invertible? If it is invertible find its inverse. If it is not invertible then find a restriction of f which is invertible. Please state the domain and range of the inverse (or local inverse) which you find.

Not 1-1 since $f(-1) = f(1)$. Restrict to $[0,1]$ and consider $y = x^2 \Rightarrow x = \sqrt{y}$ since $x \in [0,1]$.

So, $f^{-1}(y) = \sqrt{y}$ is local inverse of f w.r.t $[0,1]$. We can say, $f\big|_{[0,1]}^{-1}(y) = \sqrt{y}$.

Problem 4 [4pts] Solve the inequality $x^2 - 3 < 2x$. $\Longleftrightarrow$ $x^2 - 2x - 3 < 0$

$(x-3)(x+1) < 0$



algebraic critical #'s
$x = 3$ & $x = -1$.

$\therefore$ Sol$^{\underline{n}}$ of $x^2 - 3 < 2x$ is $\boxed{(-1,3)}$.

Problem 5 [8pts] Find the domain of $f(x) = 1/\cos(x - \pi/2)$. $= \dfrac{1}{\cos x \cos(\frac{-\pi}{2}) - \sin x \sin(\frac{-\pi}{2})}$

Thus $f(x) = \dfrac{1}{\sin(x)}$ hence as $\sin(n\pi) = 0$ for $n \in \mathbb{Z}$,

$dom(f) = \{x \in \mathbb{R} \mid x \neq n\pi \text{ for some } n \in \mathbb{Z}\}$

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Problem 6 [20pts] Prove by the $\epsilon\delta$-definition of the limit that

$$\lim_{x\to -2}(x^2+3x-4) = -6.$$

$|x^2+3x-4+6| \Rightarrow$

$= |x^2+3x+2|$

$= |(x+2)(x+1)|$

$< \delta|x+1| < 2\delta$

$-\delta < x+2 < \delta$

$\delta \le 1$ $-1 < x+2 < 1$

$-2 < x+1 < 0$

$|x+1| \le 2.$

use

$\delta = \min(1, \epsilon/2).$

Let $\epsilon > 0$ choose $\delta = \min(1, \epsilon/2) > 0$. Suppose $x \in \mathbb{R}$ and $0 < |x+2| < \delta$. Since $|x+2| < \delta \le 1$ we find $-1 < x+2 < 1$ hence $-2 < x+2 < 0$ so $|x+2| < 2$. Consider,

$$\begin{aligned} |x^2+3x-4+6| &= |x^2+3x+2| \\ &= |(x+2)(x+1)| \\ &= |x+2|\cdot|x+1| \\ &< (\delta)(2) \\ &\le \left(\frac{\epsilon}{2}\right)(2) \\ &= \epsilon. \end{aligned}$$

Hence, for each $\epsilon > 0$ we find $0 < |x+2| < \delta$ $\Rightarrow |x^2+3x-4+6| < \epsilon$. Thus, by def$^{\underline{n}}$,

$$\lim_{x\to -2}(x^2+3x-4) = -6$$

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Problem 7 [3pts] Find all real solutions of $x^3 = x$.

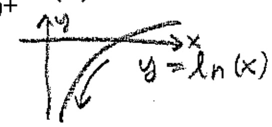
$$x^3 - x = 0 \Rightarrow (x^2-1)x = 0 \Rightarrow (x+1)(x-1)x = 0 \quad \therefore \boxed{x = 1, -1, 0}$$

Problem 8 [4pts] Suppose the tangent line to $y = f(x)$ at $x = 2$ has equation $y = 42 + 13(x-2)$. Fill in the blanks:

$f(2) =$ __42__. $f'(2) =$ __13__.

Problem 9 [30pts] Find the following limits, if they exist. If they don't exist you may simply state $\pm\infty$, *d.n.e.* or similar brief description of the divergent nature of the given limit.

(*a*.) $\lim_{x\to 0+} \ln(x) = -\infty$



(*b*.) $\lim_{x\to 3+} \dfrac{-2}{(3-x)^2} \approx \dfrac{-2}{(\delta)^2} \ll 0$ for $\delta \approx 0$

$x \approx 3+\delta$

$x - 3 \approx \delta$

$\Rightarrow \boxed{-\infty}$

(*c*.) $\lim_{x\to 5} \dfrac{\sqrt{x+11}-4}{x-5}$

$$= \lim_{x\to 5}\left[\frac{(\sqrt{x+11}-4)(\sqrt{x+11}+4)}{(x-5)(\sqrt{x+11}+4)}\right]$$

$$= \lim_{x\to 5}\left[\frac{x+11-16}{(x-5)(\sqrt{x+11}+4)}\right]$$

$$= \lim_{x\to 5}\left[\frac{\cancel{x-5}}{\cancel{(x-5)}(\sqrt{x+11}+4)}\right]$$

$$= \frac{1}{\sqrt{16}+4} = \boxed{\frac{1}{8}}$$

(*d*.) $\lim_{x\to 2} \dfrac{x-2}{x^3-2x^2+2x-4}$

$$= \lim_{x\to 2}\left[\frac{\cancel{x-2}}{\cancel{(x-2)}(x^2+2)}\right]$$

use factor by grouping or long ÷ to see it

$$= \frac{1}{4+2}$$

$$= \boxed{\frac{1}{6}}$$

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(e.) $\lim_{h\to 0} \frac{2(1+h)^2+3-5}{h}$ $= \lim_{h\to 0}\left[\frac{2(1+2h+h^2)-2}{h}\right]$

$$= \lim_{h\to 0}\left[\frac{4h+2h^2}{h}\right]$$

$$= \lim_{h\to 0}\left[4+2h\right]$$

$$= \boxed{4.}$$

Problem 10 [5pts] Given the position $s(t) = 2t^2 + 3$ calculate the velocity at time $t = 1$ (hint: may use previous calculations on test).

$$V(1) = \lim_{h\to 0}\left[\frac{s(1+h)-s(1)}{h}\right] = \lim_{h\to 0}\left[\frac{2(1+h)^2+3-5}{h}\right] = \boxed{4.}$$

(by 9e.)

Problem 11 [10pts] Show f defined below is discontinuous at $x = 2$.

$$f(x) = \begin{cases} 3|x-2|(x-2)^{-1} & if\ x < 2 \\ 3 & if\ x \geq 2 \end{cases}$$

$$\lim_{x\to 2^-}(f(x)) = \lim_{x\to 2^-}\left[\frac{3|x-2|}{x-2}\right] = \lim_{x\to 2^-}\left[\frac{-3\cancel{(x-2)}}{\cancel{x-2}}\right] = -3.$$

$$\lim_{x\to 2^+}(f(x)) = \lim_{x\to 2^+}(3) = 3 \neq -3$$

$\therefore \lim_{x\to 2} f(x) \neq f(2)$ hence f discont. at $x = 2$.

Problem 12 [10pts] Suppose that $10 + \ln(5-x) \leq f(x) \leq 3\sqrt{x} + x$ for $3 < x < 4.5$. Calculate $\lim_{x\to 4} f(x)$.

Note: $\lim_{x\to 4}(10 + \ln(5-x)) = 10 - \ln(5-4) = 10.$

$\lim_{x\to 4}(3\sqrt{x} + x) = 3\sqrt{4} + 4 = 10.$

Thus, by Squeeze Th$^{\underline{m}}$ $\underline{\lim_{x\to 4} f(x) = 10}$.

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Problem 13 [3pts] Let f be a function. State the definition of continuity at a point $a \in int(dom(f))$ (recall *int* indicates the interior, or inside of the set).

$$\lim_{x \to a} f(x) = f(a).$$

Problem 14 [12pts] Find a value for c such that function below is continuous at $x = 2$. Explain your choice in terms of arguments with limits.

$$f(x) = \begin{cases} cx - 1 & if\ x \leq 2 \\ \frac{4c(x-2)}{x^2-4} & if\ x > 2 \end{cases}$$

Note: $\lim_{x \to 2^-} [f(x)] = \lim_{x \to 2^-} (cx - 1) = 2c - 1.$

Also, $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left[\frac{4c(x-2)}{x^2-4}\right]$

$$= \lim_{x \to 2^+} \left[\frac{4c\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}\right]$$

$$= \lim_{x \to 2^+} \left[\frac{4c}{x+2}\right]$$

$$= \frac{4c}{4} = c.$$

Need $2c - 1 = c$ for $\lim_{x \to 2} f(x)$ to exist.

$\Rightarrow \boxed{c = 1}$

Problem 15 [5pts] Use the intermediate value theorem to prove that there exists a solution to the equation below: hint: think about the interval $[0, 4]$.

$$\underbrace{\sqrt{4x} - \sqrt[3]{x/4}}_{f(x)} = 1$$

$\to f(0) = 0$

$\to f(4) = \sqrt{16} - \sqrt[3]{1} = 3.$

Note, f continuous, $0 < 1 < 3$ hence, by IVT. $\exists\, c \in (0, 4)$ such that $f(c) = 1$.

$\therefore \underline{\sqrt{4c} - \sqrt[3]{c/4} = 1}.$

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Problem 16 [10pts] Let $f(x) = 1/x$. Show, by the definition, that $f'(x) = -1/x^2$ for $x \neq 0$.

$$f'(x) = \lim_{h \to 0} \left[\frac{\frac{1}{x+h} - \frac{1}{x}}{h} \right]$$

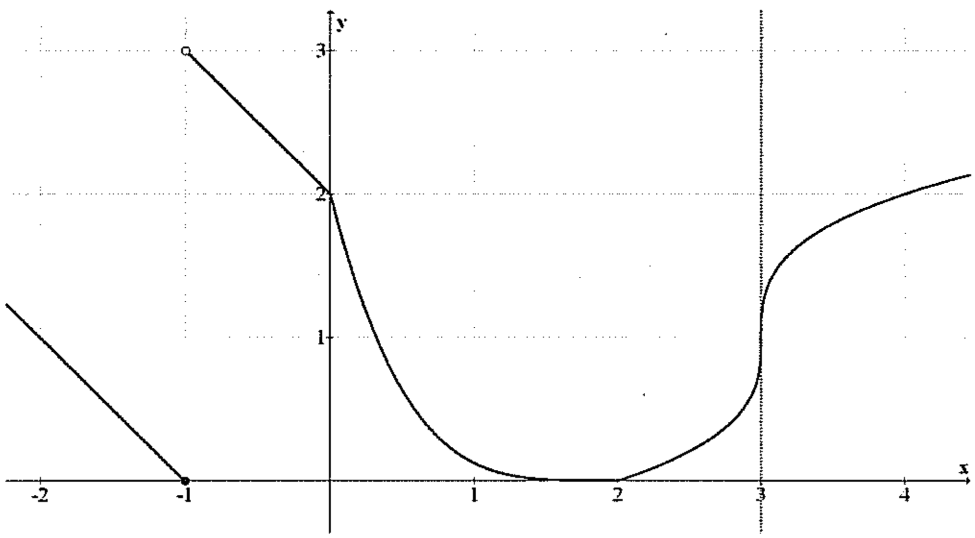
$$= \lim_{h \to 0} \left[\frac{1}{h} \frac{x-(x+h)}{x(x+h)} \right]$$

$$= \lim_{h \to 0} \left[\frac{1}{h} \frac{-h}{x(x+h)} \right]$$

$$= \lim_{h \to 0} \left[\frac{-1}{x(x+h)} \right]$$

$$= \underline{\frac{-1}{x^2}}.$$

Problem 17 [8pts] Read the derivatives from the graph of $y = f(x)$ given below, if possible. If not possible briefly explain why.



(a.) $f'(-1)$ d.n.e — f discontinuous at $x=-1$

(b.) $f'(0)$ d.n.e — kink in graph.

(c.) $f'(1.7) = 0$ — horizontal tangent

(d.) $f'(3)$ d.n.e. — vertical tangent.

Problem 18 [2pts] State the defining formula: $\cosh(x) =$ $\underline{\frac{1}{2}(e^x + e^{-x})}$.

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