

Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Thanks. You are allowed the use of a scientific (non-graphing) calculator. No electronic communication devices of any kind permitted, no IPODs, Zunes, Walkmans etc... This is a timed test and time is likely to be an issue for you, budget your time wisely. There are at least 150pts to earn on this exam.

Problem 1 [7pts] Let $f(x) = x^2 + x$. Calculate:

$$(a.) f'(x) = \underline{\hspace{2cm}}, \quad (b.) \frac{d}{dx}[f'(x)] = \underline{\hspace{2cm}}.$$

Problem 2 [63pts] Complete the indicated derivatives. If more than one variable is present in an expression you may assume that the variables are independent.

$$(a.) \frac{d}{dx} \left[\frac{x^2 + 3x + 7}{\sqrt{x}} + \pi^3 \right]$$

$$(b.) \frac{d}{db} [ax^2 + bx + c]$$

$$(c.) \frac{d}{dx} \left[(1-3x)^6 (x^2+x)^3 \right]$$

$$(d.) \frac{d}{dx} \left[\sqrt{3-x} \right]$$

$$(e.) \frac{d}{dx} \left[\sqrt[3]{x + \sqrt{4x^2}} \right]$$

$$(f.) \frac{d}{dx} \left[\ln(Ae^{Bx+C}) \right]$$

$$(g.) \frac{d}{dx} \left[\sin[x^3 + 2 \cos(7x)] \right]$$

$$(h.) \frac{d}{dx} \left[\sin^{-1}(3e^x) \right]$$

$$(i.) \frac{d}{dx} \left[\frac{\cot(x)}{x - e^{-2x}} \right]$$

Problem 3 [10pts] Let $a > 0$ with $a \neq 1$. Show that $\frac{d}{dx}[a^x] = \ln(a)a^x$.

Problem 4 [10pts] You are given standard trigonometric identities and $\lim_{h \rightarrow 0} \left[\frac{\sin(h)}{h} \right] = 1$ and $\lim_{h \rightarrow 0} \left[\frac{\cos(h) - 1}{h} \right] = 0$. **Prove that** $\frac{d}{dx} [\sin(x)] = \cos(x)$.

Problem 5 [5pts] Given $y = x + [\cos(x)]^{\sin(x)}$ calculate dy/dx .

Problem 6 [10pts] Calculate dy/dx via the technique of logarithmic differentiation (do not simplify answer and you may use y in your answer)

$$y = \frac{3e^{2x+1}(x+1)(x^2-3)^7}{\sqrt{1+\sin(x)}}$$

Problem 7 [10pts] Suppose $(x+y)^3 = y^3 + 2x + 17$.

- (a.) Calculate dy/dx via the technique of implicit differentiation.
- (b.) find the tangent line at $(1, 2)$.

Problem 8 [5pts] Approximate $\sqrt[4]{17}$ by considering the linearization of $f(x) = \sqrt[4]{x}$ centered at $a = 16$.

Problem [2pts] Suppose the tangent line to $y = f(x)$ at $x = 2$ has equation $y = 42 + 13(x - 2)$. Find the equation of the normal line to $y = f(x)$ at $x = 2$.

Problem 9 [5pts] Argue for or against the following claim: " f is a differentiable function on \mathbb{R} " where f is given as follows:

$$f(x) = \begin{cases} \cos(x) + 1 & \text{if } x < \pi/2 \\ \cos(x) - 1 & \text{if } x \geq \pi/2 \end{cases}$$

Problem 10 [5pts] Suppose some guy named Frank tells you that $P = mv$. If you are given that $\frac{dP}{dt} = 10$ and $\frac{dv}{dt} = 3$ when $m = 1$ and $v = 2$ then at what rate is m changing?

Problem 11 [15pts] Suppose that three sticks are joined to make a triangle. Furthermore, suppose these three sticks of lengths A, B, C are placed on a flat surface. Label the angles of this triangle by α, β, γ which are opposite sides A, B, C respectively. You observe that α is increasing at rate of 1 degree per second and β is decreasing at a rate of 3 degrees per second. **Find the rate of change of the area in this triangle.**

IGNORE THIS, THE PROBLEM AS STATED IS IMPOSSIBLE

Problem 12 [5pts] If a function f is continuously differentiable at $a \in \text{dom}(f)$ such that $f'(a) \neq 0$ then what does that tell you about the function? What interesting result follows given this data?

Problem 13 [5pts] Let p be a polynomial function of degree n for some $n > 1$. Define $f(x) = p(x)e^{cx}$ where c is a constant. Discuss how many horizontal tangents are possible for f on \mathbb{R} .

Problem [10pts] Suppose that $u \in C^2(\mathbb{R})$ in this problem. Suppose $g(u) = \sinh(u^2)$. Define $h(x) = g(u(x))$.

1. calculate $\frac{dh}{dx}$
2. calculate $h''(x)$
3. does $y = h(x)$ have any horizontal tangents?