

Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Thanks. You are allowed the use of a scientific (non-graphing) calculator. No electronic communication devices of any kind permitted, no IPODs, Zunes, Walkmans etc... This is a timed test and time is likely to be an issue for you, budget your time wisely. There are at least 150pts to earn on this exam.

Problem 1 [9pts] State the following theorems, make sure to include the preconditions which are necessary for the theorem to hold true.

1. Rolle's Theorem:

2. The Mean Value Theorem:

3. The Extreme Value Theorem:

Problem 2 [11pts] Show that $2x - 1 - \sin(x) = 0$ has exactly one real root.

Problem 3 [15pts] Let $f(x) = \ln(x^2 + 4x + 5)$. Locate all critical numbers for f and use the second derivative test to find the local extrema for the function.

Problem 4 [15pts] Find the intervals of increase and decrease for the function $f(x) = x + \frac{9}{x}$. Also, classify any local extreme values by applying the first derivative test.

Problem 5 [15pts] Determine where $f(x) = x^4 - 2x^3 - 36x^2$ is concave up (CU) or concave down (CD). Also, find any inflection points.

Problem 6 [15pts] Use calculus to find the point on the line $y = 3x - 7$ which is closest to $(100, 20)$. Your answer should include some indication as to how calculus shows your result is the minimum distance possible to the given line from the given point.

Problem 7 [15pts] Suppose 300 *ft* of fence is used to construct a rectangular pen for some sheep. One side of the pen doesn't need a fence because there is a steep mountain which the sheep can climb. **Use calculus to find the dimensions for the fence which maximize the area enclosed.**

(please write limits where they belong, please don't make me take points off for this again... I want you to keep your points, I already have enough)

Problem 8 [20pts] Calculate the following limits using algebra or intuition.

(a.) $\lim_{x \rightarrow \infty} \cos(x)$

(b.) $\lim_{x \rightarrow \infty} (2 + \tan^{-1}(x))$

(c.) $\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 + 2}}{x + 1}$

(d.) $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x)$

Problem 9 [15pts] Calculate the following limits using L'Hospital's rule where appropriate. Indicate usage of the rule either with the notation we used in lecture or in words to the side.

(a.) $\lim_{x \rightarrow -\infty} xe^x$

(b.) $\lim_{x \rightarrow 0} [x^2 e^{1/x^2}]$

(c.) $\lim_{x \rightarrow 0} \left[\frac{\sin(ax)}{\sin(bx)} \right]$

(Let a, b be nonzero constants.)

Problem 10 [10pts] Find local extrema for $f(x) = \frac{1}{x-1} - \frac{1}{x-3} + 1$ and also find the horizontal asymptotes for this function. Sketch the graph of $y = f(x)$.

Problem 11 [10pts] Let $f(x) = \sinh(x)$. Find intervals of increase, decrease and intervals of concave up/down. Use all of this information to graph $y = f(x)$.

Problem 12 [5pts] Suppose a smooth function f has critical points at $x = -1$ and $x = 2$. Furthermore, suppose that $f''(x) < 0$ for $x > 1$ and $f''(x) > 0$ for $x < 1$.

1. what is $f''(1)$ and how do you know that to be true from the given information?
2. find the intervals of increase and decrease for f

Problem 13 [5pts] The total energy of a free particle is a function of velocity v is given by the formula below:

$$E(v) = \frac{m_0 c^2}{\sqrt{c^2 - v^2}}$$

where $m_0, c > 0$ are constant with respect to v . **Find the minimum and maximum energy if possible. (if only one exists then use calculus to find it and to prove that is is an extreme value for the energy function).**

Problem 14 [5pts] Let p be a polynomial function of degree n for some $n > 1$. Define $f(x) = p(x)e^{cx}$ where c is a constant. Discuss how many inflection points are possible for f on \mathbb{R} .