Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Thanks. Same instructions as previous tests. There are at least 150pts to earn on this exam.

Problem 1 [7pts] Suppose $f'(x) = x + e^x$ and $f(0) = 0$. Calculate $f(x)$.

Problem 2 [7pts] Calculate $\frac{dg}{dx}$ where g is the function defined by

$$g(x) = \int_{x^2}^{\sin(x)} \sqrt[3]{t^2 + \sqrt{t}}\, dt.$$

Problem 3 [7pts] Suppose that $\int_0^1 f(x)\, dx = 3$ and suppose g is continuous with $1 \leq g(x) \leq 2$ for all $x \in [0, 1]$. Given this information find the maximum and minumum values possible for $\int_0^1 [g(x) + f(x)]\, dx$.

Problem 4 [7pts] Use the definition of the definite integral and the FTC part II to calculate the following infinite sum:

$$\lim_{n\to\infty} \sum_{i=0}^{n} \left[2 + \frac{3i}{n} \right]^2 \frac{3}{n}.$$

Problem 5 [10pts] Integrate. These are all solvable with algebra, basic identities and the basic antiderivatives. (shouldn't need a u-substitution here)

$$\int \left[\sqrt{x} + 2^x \right] dx$$

$$\int \tan^2(y)\, dy$$

$$\int \frac{1}{\sqrt{1-x^2}}\, dx$$

$$\int \sec(x)\tan(x)\, dx$$

$$\int_{-3}^{-2} \frac{1}{x}\, dx$$

Problem 6 [5pts] Integrate.

$$\int_0^{2\pi} |\sin(x)|\, dx$$

Problem 7 [5pts] Suppose $\frac{dg}{dt}$ gives the rate at which cats jump out a window. What does $\int_1^4 \frac{dg}{dt} dt$ represent given that $t = 0$ corresponds to noon and t is in hours.

Problem 8 [50pts] Integrate. Show work where necessary. Indicate all u-substitutions either implicitly or explicitly.

$$\int_{-3}^{-2} (x+3)^8 \, dx$$

$$\int \frac{\cos(\ln(x))}{x} \, dx$$

assume $0 < a < b$,

$$\int_{a^2}^{b^2} \frac{\cos(\sqrt{t})}{\sqrt{t}} \, dt$$

$$\int \frac{x+1}{2x+3}\,dx$$

$$\int \cos^3(x)\,dx$$

Problem 9 [10pts] Suppose that the acceleration of Herbert is given as a function of time t to be $a(t) = t$. Furthermore, Herbert undergoes one-dimensional motion along the x-axis where he begins at the origin with a velocity $v(0) = 1$. **Calculate** the velocity and position of Herbert as a function of time.

Problem 10 [15pts] Calculate area bounded between $y = x$ and $y = x^3$. Include a graph which indicates the typical infinitesimal region and include algebra which justifies the bounds of the integration you did to calculate the area. (in short, show your work please)

Problem 11 [15pts] Calculate area bounded between $x = 2 - y^2$ and $x = -2$. Include a graph which indicates the typical infinitesimal region and include algebra which justifies the bounds of the integration you did to calculate the area. (in short, show your work please)

Problem 12 [7pts] Suppose $A(x) = \int_1^x f(t)\,dt$ for $x \in \mathbb{R}$. Furthermore, suppose that f is a continuous function on $\mathbb{R}$. Answer the questions below in view of the information just given: (Feel free to use any important theorems which I presented in lecture to answer the questions above. (I do mean for $k \in \mathbb{N}$ in part (3.))).

1. if f is differentiable on $\mathbb{R}$ then is A also differentiable on $\mathbb{R}$?
2. if f is not differentiable then is A also a differentiable function ?
3. if f is k-times continuously differentiable then A is $(k+1)$-times continuously differentiable ?

Problem 13 [15pts] Integrate.

$$\int e^x \cosh(x)\,dx$$

$$\int \big[\, \sin(\theta)\cos(3\theta) + \cos(\theta)\sin(3\theta) \,\big]\,d\theta$$

$$\int \sin^2(\theta)\,d\theta$$

$$\int \big[\cosh(2x)\sinh(2x)\big]\,dx$$

Problem 14 [5pts] Suppose $a \neq 0$. True or False ? (explain.)

$$\int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^2}\left[\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + \frac{x}{x^2+a^2}\right] + c$$