

## § 4.9 pg. 334

$$\textcircled{1} \quad f(x) = 6x^2 - 8x + 3$$

$$\Rightarrow F(x) = \frac{6x^3}{3} - \frac{8x^2}{2} + 3x + C = 2x^3 - 4x^2 + 3x + C$$

$$\textcircled{2} \quad f(x) = 1 - x^3 + 12x^5$$

$$F(x) = x - \frac{x^4}{4} + \frac{12}{6}x^6 + C = x - \frac{x^4}{4} + 2x^6$$

$$\textcircled{3} \quad f(x) = 5x^{\frac{1}{4}} - 7x^{\frac{3}{4}}$$

$$F(x) = \frac{5x^{\frac{5}{4}}}{5/4} - \frac{7x^{\frac{7}{4}}}{7/4} + C = 4x^{\frac{5}{4}} - 4x^{\frac{7}{4}} + C$$

$$\textcircled{4} \quad f(x) = 2x + 3x^{1.7}$$

$$F(x) = x^2 + 3 \frac{x^{2.7}}{2.7} + C$$

$$\textcircled{5} \quad f(x) = \frac{10}{x^3} = 10x^{-3}$$

$$F(x) = \frac{10x^{-2}}{-2} = -5 \frac{x^{-8}}{4} + C$$

$$\textcircled{6} \quad f(x) = \sqrt[3]{x^2} - \sqrt{x^3} = x^{\frac{2}{3}} - x^{\frac{3}{2}}$$

$$F(x) = \frac{x^{\frac{5}{3}}}{5/3} - \frac{x^{\frac{5}{2}}}{5/2} + C$$

$$= \frac{3}{5}x^{\frac{5}{3}} - \frac{2}{5}x^{\frac{5}{2}} + C$$

$$\textcircled{7} \quad g(t) = \frac{t^3 + 2t^2}{\sqrt{t}} = t^{\frac{5}{2}} + 2t^{\frac{3}{2}}$$

$$G(t) = \frac{t^{\frac{7}{2}}}{7/2} + 2 \frac{t^{\frac{5}{2}}}{5/2} + C$$

$$= \frac{2}{7}t^{\frac{7}{2}} + \frac{4}{5}t^{\frac{5}{2}} + C$$

$$\textcircled{8} \quad f(t) = 3 \cos t - 4 \sin t$$

$$F(t) = 3 \sin t + 4 \cos t + C$$

$$\textcircled{9} \quad f(x) = \frac{3}{x^2} - \frac{5}{x^4} = 3x^{-2} - 5x^{-4}$$

$$F(x) = 3 \frac{x^{-1}}{-1} - 5 \frac{x^{-3}}{-3} + C = -3/x + \frac{5}{3x^3} + C$$

$$\textcircled{10} \quad f(x) = 3e^x + 7 \sec^2(x)$$

$$F(x) = 3e^x + 7 \tan(x) + C$$

$$\textcircled{11} \quad f(x) = 2x + 5(1-x^2)^{-\frac{1}{2}}$$

$$F(x) = x^2 + 5 \sin^{-1} x + C$$

(56)

$$\textcircled{12} \quad f(x) = \frac{x^2 + x + 1}{x} = x + 1 + \frac{1}{x}$$

$$F(x) = \frac{x^2}{2} + x + \ln|x| + C$$

#16 Given  $f''$  find  $f$ ,

$$f'' = 2 + x^3 + x^6$$

$$f' = 2x + \frac{x^4}{4} + \frac{x^7}{7} + C$$

$$f = x^2 + \frac{x^5}{20} + \frac{x^8}{56} + CX + d$$

just anti-differentiate twice.

Remark: Given  $f(x)$  continuous we can find the antiderivative  $F(x)$  with  $F'(x) = f(x)$  this is the same as finding the indefinite integral off  $\int f(x) dx = F(x)$

usually I'll use the  $\int$  notation.

② (a) approx. area under graph on pg. 355 #2 from  $x=0$  to  $x=12$ ,

$$\Delta x = \frac{b-a}{n} = \frac{12-0}{6} = 2 = \Delta x$$

$$\begin{aligned} (i) L_6 &= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] \\ &= 2[9 + 8.8 + 8.3 + 7.4 + 6 + 4] \quad \text{estimating from graph.} \\ &= 2[43.5] = 87 = L_6 \end{aligned}$$

$$\begin{aligned} (ii) R_6 &= 2[f(x_1) + f(x_2) + \dots + f(x_6)] \\ &= 2[8.8 + 8.3 + 7.4 + 6 + 4 + 1] \\ &= 2[35.5] = 71 = R_6 \end{aligned}$$

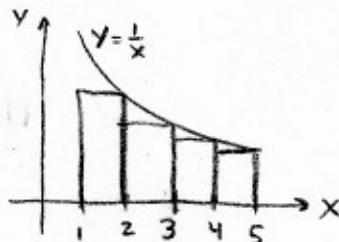
$$\begin{aligned} (iii) M_6 &= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)] \\ &= 2[9 + 8.5 + 7.8 + 6.7 + 5.1 + 2.9] \\ &= 2[40] = 80 = M_6 \end{aligned}$$

(b)  $L_6$  is an over-estimate.

(d)  $M_6$  probably best approx. the area.

(c)  $R_6$  is an under-estimate.

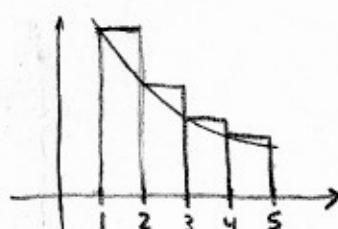
③



$$R_4 = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{30+20+15+12}{60} = \frac{77}{60} \approx 1.283$$

(underestimate)

(b)



$$L_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{60+30+20+15}{60} = \frac{125}{60} \approx 2.083$$

(over estimate)

It didn't ask but the true area is easy to calculate!

$$\int_1^5 \frac{1}{x} dx = \ln(x) \Big|_1^5 = \ln(5) \approx 1.61$$

⑬  $\int_0^6 \frac{dx}{dt} dt = \int_0^6 v dt = \text{distance traveled} = \text{area under graph. Use } L_6 \text{ to approx.}$

$$\text{distance} \approx 70 + 48 + 32 + 23 + 17 + 8 = 198 \text{ ft}$$

§ 5.2 #18  
p. 368

Express the limit as a definite integral  
on the given interval. Goal: understand def<sup>n</sup> of  $\int_a^b f(x) dx$  better.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n x_i \sin(x_i)(\Delta X) \quad \text{on } [0, \pi]$$

The definition of  $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \left( \sum_{i=1}^n f(x_i) \Delta X \right)$

where  $\Delta X = \frac{b-a}{n}$  and  $x_i \in [a + i \Delta X, a + (i+1) \Delta X]$

you can pick  $x_i$  by any # of rules left, right, midpt. etc...  
anyway just compare the  $\sum$ 's to see

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n x_i \sin(x_i) \Delta X = \boxed{\int_0^\pi x \sin(x) dx}$$

§ 5.2 #20  
p. 368

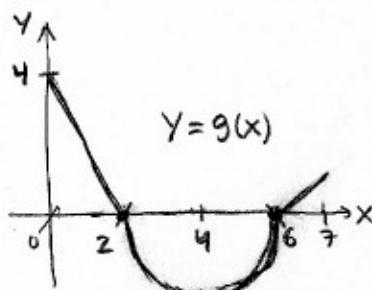
Rewrite limit as a definite integral (just like #18)

$$\lim_{n \rightarrow \infty} \left( \sum_{i=1}^n \sqrt{x_i^*} \Delta X \right) = \boxed{\int_1^4 \sqrt{x} dx}$$

(on  $[1, 4]$ )

- Question: Can you see why people call the integral a "continuous sum"?

(30)



The integral is the area under the curve, but remember it is negative when under x-axis.

$$(a) \int_0^2 g(x)dx = \frac{1}{2}(2)(4) = 4 \quad (\text{area of triangle})$$

$$(b) \int_2^6 g(x)dx = \frac{1}{2}(\pi(2)^2) = -2\pi \quad (\text{area of } \frac{1}{2} \text{ a circle})$$

negative b/c it's under x-axis.

$$(c) \int_0^7 g(x)dx = \int_0^2 g(x)dx + \int_2^6 g(x)dx + \int_6^7 g(x)dx$$

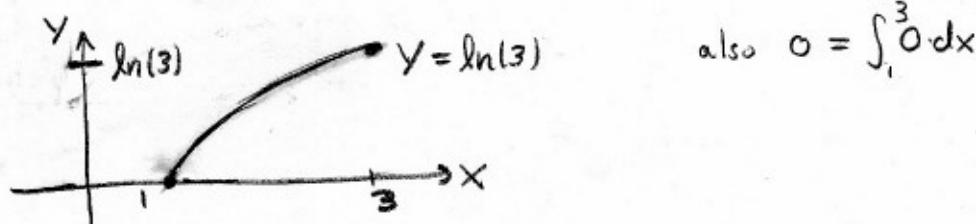
$$= 4 - 2\pi + \frac{1}{2}$$

$$\approx -1.78$$

$$(40) \int_2^{10} f(x)dx - \int_2^7 f(x)dx = \int_2^7 f(x)dx + \int_7^{10} f(x)dx - \int_2^7 f(x)dx = \boxed{\int_7^{10} f(x)dx}$$

$$(47) 2\ln(3) = \int_1^3 \ln(3)dx \quad \text{by property 1. pg. 364 also, } 0 = \int_1^3 0dx$$

So we then note  $0 \leq \ln(x) \leq \ln(3)$  for  $1 \leq x \leq 3$



So applying prop. 7 on pg. 366 twice we get

$$\int_1^3 0dx \leq \int_1^3 \ln(x)dx \leq \int_1^3 \ln(3)dx \Rightarrow \boxed{0 \leq \int_1^3 \ln(x)dx \leq 2\ln(3)}$$

(1)  $\int_5^{10} w'(t)dt$  represents  $\int_5^{10} \frac{dw}{dt} dt = \int_{w(5)}^{w(10)} dw = w(10) - w(5)$  = weight gained.

(2)  $\int_a^b I(t)dt = \int_a^b \frac{dq}{dt} dt = q(b) - q(a)$  represents total difference in charge

(3)  $\int_0^{120} r(t)dt$  represents oil leaked first 120 minutes.

(4)  $100 + \int_0^{15} n'(t)dt$  = # of bees at 15 weeks.

(5)  $\int_{1000}^{5000} R'(x)dx$  = revenue from selling the 1000<sup>th</sup> to the 5000<sup>th</sup> units.

(6)  $\int_3^5 f(x)dx = \int_3^5 \frac{dy}{dx} dx = \int_{y(3)}^{y(5)} dy = y(5) - y(3)$  total height scaled from miles 3 to 5 of the hizze.

(7)  $\int_0^{100} f(x)dx$  has units Nm. (like the work integral  $W = \int F dx$ , Nm = Joule)

(8)  $\frac{da}{dx}$  has units  $\frac{lb}{ft^2}$  while  $\int_2^8 a(x)dx$  has units  $\frac{lbf \cdot ft}{ft} = \underline{\underline{lbf}}$

These are all basic integrals, sometimes a step or two is necessary to see that though, (60)

§5.3  
P.377 (9)  $\int_{-1}^3 x^5 dx = \frac{x^6}{6} \Big|_{-1}^3 = \frac{1}{6}(3^6 - (-1)^6) \cong [121.3]$

(10)  $\int_1^2 x^{-2} dx = \frac{x^{-1}}{-1} \Big|_1^2 = \frac{-1}{x} \Big|_1^2 = \frac{-1}{2} + \frac{1}{1} = \boxed{\frac{1}{2}}$

(11)  $\int_2^8 (4x+3) dx = \left(\frac{4x^2}{2} + 3x\right) \Big|_2^8 = [2(8)^2 + 3(8)] - [2(2)^2 + 3(2)] = \boxed{138}$

(12)  $\int_0^4 (1+3y-y^2) dy = \left(y + \frac{3y^2}{2} - \frac{y^3}{3}\right) \Big|_0^4 = 4 + \frac{3(4)^2}{2} - \frac{4^3}{3} \cong \boxed{6.67}$

(13)  $\int_0^4 \sqrt{x} dx = \int_0^4 x^{1/2} dx = \frac{2}{3}x^{3/2} \Big|_0^4 = \frac{2}{3}(4)^{3/2} = \boxed{\frac{16}{3}}$

(14)  $\int_{\pi}^{2\pi} \cos(\theta) d\theta = \sin\theta \Big|_{\pi}^{2\pi} = \sin(2\pi) - \sin(\pi) = \boxed{0}$

(15)  $\int_{-1}^0 (2x-e^x) dx = 2 \int_{-1}^0 x dx - \int_{-1}^0 e^x dx = 2 \frac{x^2}{2} \Big|_{-1}^0 - e^x \Big|_{-1}^0 = -1 - e^0 + e^{-1} = \boxed{e^{-1} - 2}$

(16)  $\int_0^1 x^{3/7} dx = \frac{7}{10}x^{10/7} \Big|_0^1 = \boxed{\frac{7}{10}}$

(17)  $\int_1^2 \frac{3}{t^4} dt = 3 \int_1^2 t^{-4} dt = 3 \frac{t^{-3}}{-3} \Big|_1^2 = \frac{-1}{t^3} \Big|_1^2 = \frac{-1}{8} - \left(-\frac{1}{1}\right) = \boxed{\frac{7}{8}}$

(18)  $\int_1^4 \frac{1}{\sqrt{x}} dx = \int_1^4 x^{-1/2} dx = 2x^{1/2} \Big|_1^4 = 2\sqrt{4} - 2\sqrt{1} = \boxed{2}$

(19)  $\int_1^2 \frac{x^2+1}{\sqrt{x}} dx = \int_1^2 \frac{x^2}{\sqrt{x}} dx + \int_1^2 \frac{1}{\sqrt{x}} dx = \int_1^2 x^{3/2} dx + \int_1^2 x^{-1/2} dx = \frac{2}{5}x^{5/2} \Big|_1^2 + 2x^{1/2} \Big|_1^2 \cong \boxed{2.69}$

(20)  $\int_0^2 (x^3-1)^2 dx = \int_0^2 (x^6 - 2x^3 + 1) dx = \left(\frac{x^7}{7} - \frac{2x^4}{4} + x\right) \Big|_0^2 \cong \boxed{12.29}$

(21)  $\int_{\pi/4}^{\pi/3} \sin(t) dt = -\cos(t) \Big|_{\pi/4}^{\pi/3} = -\cos(\pi/3) + \cos(\pi/4) = -\frac{1}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-1}{2} \cong \boxed{0.2071}$

(22)  $\int_1^2 \frac{4+u^2}{u^3} du = 4 \int_1^2 u^{-3} du + \int_1^2 u^{-1} du = 4 \frac{u^{-2}}{-2} \Big|_1^2 + \ln(u) \Big|_1^2 = -2\left(\frac{1}{2^2} - \frac{1}{1}\right) + \ln(2) - \ln(1) \cong \boxed{2.193}$

(23)  $\int_0^1 u(\sqrt{u} + \sqrt[3]{u}) du = \int_0^1 u^{3/2} du + \int_0^1 u^{4/3} du = \frac{2}{5}u^{5/2} \Big|_0^1 + \frac{3}{7}u^{7/3} \Big|_0^1 = \frac{2}{5} + \frac{3}{7} = \frac{14+15}{35} = \boxed{\frac{29}{35}}$

(24)  $\int_0^5 (2e^x + 4\cos(x)) dx = (2e^x + 4\sin(x)) \Big|_0^5 \cong \boxed{291}$

$$(25) \int_{\pi/6}^{\pi/3} \csc^2(\theta) d\theta = -\cot(\theta) \Big|_{\pi/6}^{\pi/3} = -\cot(\pi/3) + \cot(\pi/6) = \frac{2\sqrt{3}}{3} \approx 1.15$$

$$(26) \int_1^8 \frac{x-1}{x^3} dx = \int_1^8 (x^{1/3} - x^{-2/3}) dx = \left( \frac{3}{4} x^{4/3} - 3 x^{1/3} \right) \Big|_1^8 = \frac{83}{4} = 8.25$$

$$(27) \int_1^9 \frac{1}{2x} dx = \frac{1}{2} \int \frac{1}{x} dx = \left( \frac{1}{2} \ln(x) \right) \Big|_1^9 = \frac{1}{2} \ln(9) - \frac{1}{2} \ln(1) = \frac{1}{2} \ln(9) = \ln(3)$$

$$(28) \int_{\ln(3)}^{\ln(6)} 8e^x dx = 8e^x \Big|_{\ln(3)}^{\ln(6)} = 8e^{\ln(6)} - 8e^{\ln(3)} = 48 - 24 = 24$$

$$(29) \int_8^9 2^t dt = \frac{2^t}{\ln(2)} \Big|_8^9 = \frac{1}{\ln(2)} (2^9 - 2^8) = \frac{256}{\ln(2)} \approx 369.33$$

$$(30) \int_{\pi/3}^{\pi/2} \csc(x) \cot(x) dx = -\csc(x) \Big|_{\pi/3}^{\pi/2} = -\csc(\pi/2) + \csc(\pi/3) = 0.1547 = \frac{2\sqrt{3}}{3} - 1$$

$$(31) \int_1^{\sqrt{3}} \frac{6}{1+x^2} dx = 6 \tan^{-1}(x) \Big|_1^{\sqrt{3}} = 6 \tan^{-1}(\sqrt{3}) - 6 \tan^{-1}(1) = \frac{\pi}{2} \approx 1.571$$

$$(32) \int_0^{0.5} \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) \Big|_0^{0.5} = \sin^{-1}(0.5) - \sin^{-1}(0) = 0.524$$

$$(33) \int_0^{\pi/4} \frac{1+\cos^2 \theta}{\cos^2 \theta} d\theta = \int_0^{\pi/4} \sec^2(\theta) d\theta + \int_0^{\pi/4} d\theta = \tan(\theta) \Big|_0^{\pi/4} + \theta \Big|_0^{\pi/4} = 1 + \frac{\pi}{4} = 1.785$$

$$(34) \int_1^2 |x-x^2| dx = \int_{-1}^2 |x(1-x)| dx$$

+ + + +
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x(1-x)

↗ need this to break  
 down the absolute  
 value into cases.

$$\begin{aligned}
 &= \int_{-1}^0 -x(1-x) dx + \int_0^1 x(1-x) dx + \int_1^2 -x(1-x) dx \\
 &= - \int_1^0 (x-x^2) dx + \int_0^1 (x-x^2) dx - \int_1^2 (x-x^2) dx \\
 &= - \left( \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_{-1}^0 + \left( \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_0^1 - \left( \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_1^2 \\
 &\approx -0.8333 + 0.1666 + 0.8333 \\
 &\approx 1.833
 \end{aligned}$$

$$(41) \frac{d}{dx} (\sqrt{x^2+1} + C) = \frac{d}{dx} ((x^2+1)^{1/2}) = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2+1}} \leftarrow \begin{array}{l} \text{the integrand} \\ \text{as desired.} \end{array}$$

$$(42) \frac{d}{dx} (x \sin x + \cos x + C) = \cancel{\sin(x)} + x \cos(x) - \cancel{\sin(x)} = x \cos(x) \leftarrow \begin{array}{l} \text{the integrand} \\ \text{as we want.} \end{array}$$

(51)  $v(t) = 3t - 5 = \frac{dx}{dt} \Rightarrow \int_0^3 v(t) dt = \int_0^3 \frac{dx}{dt} dt = x(3) - x(0) \leftarrow \text{displacement}.$

(a)  $\int_0^3 v(t) dt = \int_0^3 (3t - 5) dt = \left(3\frac{t^2}{2} - 5t\right)_0^3 = \frac{27}{2} - 15 = \frac{27-30}{2} = \boxed{-\frac{3}{2} m}$

(b)  $\int_0^3 |v(t)| dt = \int_0^{5/3} (5-3t) dt + \int_{5/3}^3 (3t-5) dt \quad \text{since } |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$   
 $= \left(5t - \frac{3t^2}{2}\right)_0^{5/3} + \left(\frac{3t^2}{2} - 5t\right)_{5/3}^3$   
 $= 4.166 + 2.666 = \boxed{6.83 \text{m} = \text{distance}}$

(52)  $v(t) = t^2 - 2t - 8 = (t-4)(t+2)$

(a)  $\int_1^6 (t^2 - 2t - 8) dt = \left(\frac{t^3}{3} - t^2 - 8t\right)_1^6 = \boxed{-3.33 \text{m} = \text{displacement}}$

(b)  $\int_1^6 |t^2 - 2t - 8| dt = \int_1^4 (-t^2 + 2t + 8) dt + \int_4^6 (t^2 - 2t - 8) dt$   
 $= \left(-\frac{t^3}{3} + t^2 + 8t\right)_1^4 + \left(\frac{t^3}{3} - t^2 - 8t\right)_4^6$   
 $= 18 + 14.66 = \boxed{32.67 \text{m} = \text{distance}}$

(53)  $a(t) = t + 4$  and  $v(0) = 5$  note:  $\int_0^t a(u) du = \int_0^t \frac{dv}{du} du = v(t) - v(0) = v(t) - 5$

a.) Also note  $\int_0^t a(u) du = \int_0^t (u+4) du = \frac{u^2}{2} + 4u \Big|_0^t = \frac{1}{2}t^2 + 4t = v(t) - 5$   
 $\therefore \boxed{v(t) = \frac{1}{2}t^2 + 4t + 5}$

b.)  $x(10) - x(0) = \int_0^{10} \frac{dx}{dt} dt = \int_0^{10} \left(\frac{1}{2}t^2 + 4t + 5\right) dt = \left(\frac{1}{6}t^3 + \frac{2}{3}t^2 + 5t\right)_0^{10} = \boxed{416.7 \text{m}}$

(54)  $a(t) = 2t + 3$  and  $v(0) = -4$  again  $\int_0^t a(u) du = v(t) - v(0) = v(t) + 4$

a.)  $\int_0^t a(u) du = \int_0^t (2u+3) du = (u^2 + 3u) \Big|_0^t = \underline{t^2 + 3t} = v(t) + 4$

thus  $\boxed{v(t) = t^2 + 3t - 4}$

(b.)  $x(3) - x(0) = \int_0^3 \frac{dx}{dt} dt = \int_0^3 (t^2 + 3t - 4) dt = \left(\frac{1}{3}t^3 + \frac{3}{2}t^2 - 4t\right)_0^3 = \boxed{10.5 \text{m}}$

(pg. 386-387) § 5.4 #'s 8, 10, 13, 18

$$⑧ g'(x) = \frac{d}{dx} \int_1^x \ln(t) dt = \ln(x)$$

$$⑩ F'(x) = \frac{d}{dx} \int_x^{10} \tan \theta d\theta = -\tan(x)$$

$$⑬ \frac{dy}{dx} = \frac{d}{dx} \int_3^{\sqrt{x}} \frac{\cos t}{t} dt = \frac{\cos \sqrt{x}}{\sqrt{x}} \frac{d}{dx} (\sqrt{x}) = \frac{\cos \sqrt{x}}{\sqrt{x}} \frac{1}{2\sqrt{x}} = \frac{\cos \sqrt{x}}{2x}$$

We used the formula derived in class for 8, 10 & 13

$$\frac{d}{dx} \int_{h(x)}^{g(x)} f(u) du = f(g(x)) g'(x) - f(h(x)) h'(x)$$

You should be able to derive this formula from basic principles.

⑯ Find the interval on which the curve  $y = \int_0^x \frac{1}{1+t+t^2} dt$  is concave upward.  
We know that it is concave up when  $y'' > 0$  so we must find  $y''$ ,

$$y' = \frac{dy}{dx} = \frac{d}{dx} \left( \int_0^x \frac{1}{1+t+t^2} dt \right) = \frac{1}{1+x+x^2}$$

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dx} \left( \frac{1}{1+x+x^2} \right) = \frac{-1}{(1+x+x^2)^2} \frac{d}{dx}(1+x+x^2) = \frac{-1-2x}{(1+x+x^2)^2}$$

A quick check of the quadratic formula will tell us if the denominator of  $y''$  is ever zero:  $1+x+x^2=0 \Leftrightarrow x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2} \leftarrow \text{complex}$

So the denominator is always positive in  $y''$ , we only need to worry about the numerator

$$-1-2x > 0 \Rightarrow -1 > 2x \Rightarrow x < -\frac{1}{2}$$

Or use a number line if you like  $-1-2x=0$  when  $x = -\frac{1}{2}$

$$\begin{array}{c} + + + + + + + \\ \hline - - - - - - - \end{array} \quad y'' = \frac{-1-2x}{(1+x+x^2)^2}$$

$$\textcircled{2} \quad \int x(4+x^2)^{10} dx = \int u^{10} \times \frac{du}{2x}$$

$$\begin{cases} u = 4+x^2 \\ du = 2x dx \\ dx = \frac{du}{2x} \end{cases} \quad = \frac{1}{2} \int u^{10} du$$

$$= \frac{1}{2} \frac{u^{11}}{11} + C$$

$$= \boxed{\frac{1}{22}(4+x^2)^{11} + C}$$

$$\textcircled{4} \quad \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = \int \sin(u) \frac{1}{\sqrt{x}} 2\sqrt{x} du$$

$$\begin{cases} u = \sqrt{x} \\ \frac{du}{dx} = \frac{1}{2\sqrt{x}} \\ dx = 2\sqrt{x} du \end{cases} \quad = \int 2 \sin(u) du$$

$$= -2 \cos(u) + C$$

$$= \boxed{-2 \cos(\sqrt{x}) + C}$$

$$\textcircled{6} \quad \int e^{\sin \theta} \cos \theta d\theta = \int e^u du$$

$$\begin{cases} u = \sin \theta \\ du = \cos \theta d\theta \end{cases} \quad = e^u + C$$

$$= \boxed{e^{\sin \theta} + C}$$

$$\textcircled{8} \quad \int x e^{x^2} dx = \int e^u x \cdot \frac{du}{2x}$$

$$\begin{cases} u = x^2 \\ du = 2x dx \\ dx = \frac{du}{2x} \end{cases} \quad = \frac{1}{2} \int e^u du$$

$$= \frac{1}{2} e^u + C$$

$$= \boxed{\frac{1}{2} e^{x^2} + C}$$

$$\textcircled{12} \quad \int (2-x)^6 dx = \int u^6 (-du)$$

$$\begin{cases} u = 2-x \\ du = -dx \end{cases} \quad = - \int u^6 du$$

$$= -\frac{u^7}{7} + C$$

$$= \boxed{-\frac{1}{7}(2-x)^7 + C}$$

$$\textcircled{14} \quad \text{Let } u = x^2 + 1 \text{ then } \frac{du}{dx} = 2x \therefore dx = \frac{du}{2x}$$

$$\int \frac{x dx}{x^2+1} = \int \frac{x}{u} \frac{du}{2x} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln|u| + C$$

$$\therefore \int \frac{x dx}{x^2+1} = \boxed{\frac{1}{2} \ln|x^2+1| + C}$$

$$\textcircled{16} \quad \text{Let } u = 1-t^3, \frac{du}{dt} = -3t^2$$

$$\text{then } dt = \frac{du}{-3t^2}$$

$$\int t^2 \cos(1-t^3) dt = \int t^2 \cos(u) \frac{du}{-3t^2}$$

$$= -\frac{1}{3} \int \cos(u) du$$

$$= -\frac{1}{3} \sin(u) + C$$

$$= \boxed{-\frac{1}{3} \sin(1-t^3) + C}$$

$$\textcircled{18} \quad \text{Try } u = 3-5y \Rightarrow \frac{du}{dy} = -5 \therefore dy = \frac{du}{-5}$$

$$\int \sqrt[3]{3-5y} dy = \int \sqrt[3]{u} \frac{du}{-5}$$

$$= -\frac{1}{5} \left( u^{\frac{4}{3}} / \left( \frac{4}{3} \right) \right) + C$$

$$= \boxed{-\frac{3}{20} (3-5y)^{\frac{4}{3}} + C}$$

$$\textcircled{20} \quad u = \tan^{-1}(x) \Rightarrow \frac{du}{dx} = \frac{1}{1+x^2}$$

$$\text{So } dx = (1+x^2) du$$

$$\int \frac{\tan^{-1}(x) dx}{1+x^2} = \int \frac{u}{1+x^2} (1+x^2) du$$

$$= \int u du$$

$$= \frac{1}{2} u^2 + C$$

$$= \boxed{\frac{1}{2} (\tan^{-1}(x))^2 + C}$$

$$\textcircled{22} \quad \text{Let } u = \sin x \Rightarrow du = \cos(x) dx$$

$$\int \cot(x) dx = \int \frac{\cos(x) dx}{\sin(x)} = \int \frac{du}{u} = \boxed{\ln|\sin(x)| + C}$$

$$\textcircled{24} \quad \text{Let } u = \frac{\pi}{x} \Rightarrow \frac{du}{dx} = -\frac{\pi}{x^2} \therefore dx = -\frac{x^2}{\pi} du$$

$$\int \frac{\cos(\frac{\pi}{x}) dx}{x^2} = \int \frac{\cos u}{x^2} \left( -\frac{x^2}{\pi} du \right)$$

$$= -\frac{1}{\pi} \int \cos u du$$

$$= \boxed{-\frac{1}{\pi} \sin(\frac{\pi}{x}) + C}$$

$$\textcircled{26} \quad u = \cos(x) \Rightarrow du = -\sin(x) dx$$

$$\int \frac{\sin(x) dx}{1+\cos^2 x} = - \int \frac{du}{1+u^2}$$

$$= -\tan^{-1}(u) + C$$

$$= \boxed{-\tan^{-1}(\cos(x)) + C}$$

$$\#25 \quad u = \cot(x) \Rightarrow \frac{du}{dx} = -\csc^2(x)$$

Then  $\csc^2(x) dx = -du$

$$\int \sqrt{\cot(x)} \csc^2(x) dx = - \int \sqrt{u} du$$

$$= -\frac{2}{3} u^{3/2} + C$$

$$= \boxed{-\frac{2}{3} (\cot(x))^{3/2} + C}$$

(#31)

$$\int \frac{1+x}{1+x^2} dx = \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx$$

BASIC INTEGRAL

use u-subst.

$$u = 1+x^2$$

$$\frac{du}{2} = x dx$$

$$\int \frac{x dx}{1+x^2} = \int \frac{du}{2u}$$

$$= \frac{1}{2} \ln|u| + C$$

$$= \frac{1}{2} \ln|1+x^2| + C_1$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C_2, \quad C_1 + C_2 = C$$

$$\Rightarrow \boxed{\int \frac{1+x}{1+x^2} dx = \tan^{-1}(x) + \frac{1}{2} \ln|1+x^2| + C}$$

(#41) We can do these 2-ways.

$$u = \pi t \Rightarrow du = \pi dt \Rightarrow dt = \frac{du}{\pi}$$

$$\begin{aligned} \int_0^1 \cos(\pi t) dt &= \int_{u(0)}^{u(1)} \cos(u) \frac{du}{\pi} \\ &= \frac{1}{\pi} \sin(u) \Big|_{u(0)}^{u(1)} \\ &= \frac{1}{\pi} \sin(\pi t) \Big|_0^1 \\ &= \frac{1}{\pi} (\sin \pi - \sin 0) = 0. \end{aligned}$$

Or rather than substituting back in the x-values & expressions in the last two lines you can instead use

$$u(1) = \pi(1) = \pi$$

$$u(0) = \pi(0) = 0$$

And evaluate line 2 directly

$$\int_0^1 \cos(\pi t) dt = \frac{1}{\pi} (\sin(\pi) - \sin(0)) = 0.$$