

Homework 31, Calculus I

①

§5.5 # 35

$$\begin{aligned} \int_0^2 (x-1)^{25} dx &= \frac{1}{26} u^{26} \Big|_{-1}^1 \\ &= \frac{1}{26} (1 - 1) \\ &= \boxed{0} \end{aligned}$$

$$\begin{aligned} u &= x-1 \\ du &= dx \\ u(2) &= 2-1=1 \\ u(0) &= 0-1=-1 \end{aligned}$$

§5.5 # 37

$$\begin{aligned} \int_0^1 x^2 (1+2x^3)^5 dx &= \int_1^3 \frac{1}{6} u^5 du \\ &= \frac{1}{36} u^6 \Big|_1^3 \\ &= \frac{1}{36} (3^6 - 1) \\ &= \frac{728}{36} \\ &= \boxed{\frac{182}{9}} \end{aligned}$$

$$\begin{aligned} u &= 1+2x^3 \\ du &= 6x^2 dx \\ u(0) &= 1+2(0) = 1 \\ u(1) &= 3 \end{aligned}$$

§5.5 # 39

$$\begin{aligned} \int_0^\pi \sec^2(t/4) dt &= 4 \tan(t/4) \Big|_0^\pi \\ &= 4 \tan(\pi/4) - 4 \tan(0) \\ &= \boxed{4} \end{aligned}$$

§5.5 # 41

$$\begin{aligned} \int_{-\pi/6}^{\pi/6} \tan^3 \theta d\theta &= \int_{-\pi/6}^{\pi/6} (\sec^2 \theta - 1) \tan \theta d\theta \\ &= \int_{-\pi/6}^{\pi/6} \sec \theta \sec \theta \tan \theta d\theta - \int_{-\pi/6}^{\pi/6} \tan \theta d\theta \\ &= \int_{-2/\sqrt{3}}^{2/\sqrt{3}} u du - \ln |\sec \theta| \Big|_{-\pi/6}^{\pi/6} \\ &= \frac{1}{2} \left(\left(\frac{2}{\sqrt{3}} \right)^2 - \left(\frac{-2}{\sqrt{3}} \right)^2 \right) - \ln \left| \frac{2}{\sqrt{3}} \right| + \ln \left| \frac{2}{\sqrt{3}} \right| = \boxed{0} \end{aligned}$$

$$\begin{aligned} u &= \sec \theta \\ du &= \sec \theta \tan \theta d\theta \\ u(\pi/6) &= \sec(\pi/6) \\ &= \frac{1}{\cos(\pi/6)} \\ &= \frac{2}{\sqrt{3}} \end{aligned}$$

§5.5#43

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_1^{27} \frac{1}{2} u^{-2/3} du$$

$$= \frac{3}{2} u^{1/3} \Big|_1^{27}$$

$$= \frac{3}{2} (\sqrt[3]{27} - \sqrt[3]{1})$$

$$= \boxed{3}$$

②

$$\begin{aligned} u &= 1+2x & u(13) &= 27 \\ du &= 2dx & u(0) &= 1 \\ dx &= \frac{1}{2} du \end{aligned}$$

$$\sqrt[3]{27} = 3$$

§5.5#45

$$\int_0^a x \sqrt{x^2+a^2} dx = \int_{a^2}^{2a^2} \frac{1}{2} u^{1/2} du$$

$$= \frac{1}{3} u^{3/2} \Big|_{a^2}^{2a^2}$$

$$= \frac{1}{3} (2a^2)^{3/2} - \frac{1}{3} (a^2)^{3/2}$$

$$= \boxed{\frac{1}{3} (2^{3/2} - 1) a^3}$$

$$\begin{aligned} u &= x^2+a^2 & u(a) &= 2a^2 \\ du &= 2x dx & u(0) &= a^2 \\ x dx &= \frac{1}{2} du \end{aligned}$$

§5.5#47

$$\int_1^2 x \sqrt{x-1} dx = \int_0^1 (u+1) \sqrt{u} du$$

$$= \int_0^1 (u^{3/2} + u^{1/2}) du$$

$$= \frac{2}{5} + \frac{2}{3}$$

$$= \boxed{16/15}$$

$$\begin{aligned} u &= x-1 & u(2) &= 1 \\ du &= dx & u(1) &= 0 \\ x &= u+1 \end{aligned}$$

§5.5#49

$$\int_{1/2}^1 \cos(x^{-2}) \frac{dx}{x^3} = \int_4^1 \frac{-1}{2} \cos(u) du$$

$$= \frac{-1}{2} \sin(u) \Big|_4^1$$

$$= \boxed{\frac{1}{2} (\sin(4) - \sin(1))}$$

$$\begin{aligned} u &= x^{-2} & u(1) &= 1 \\ du &= \frac{-2}{x^3} dx & u(1/2) &= (\frac{1}{2})^{-2} = 4 \\ -\frac{1}{2} du &= \frac{dx}{x^3} \end{aligned}$$