Homework 36, Calculus I

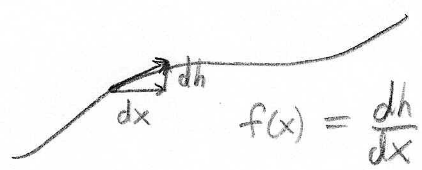
§5.4 #41 use $|x| = \begin{cases} x & x>0 \\ -x & x<0 \end{cases}$

$$\int_{-1}^{2} (x - 2|x|)\,dx = \int_{-1}^{0}(x+2x)\,dx + \int_{0}^{2}(x-2x)\,dx$$
$$= \frac{3}{2}x^2\Big|_{-1}^{0} - \frac{1}{2}x^2\Big|_0^2$$
$$= \frac{3}{2}(0-1) - \frac{1}{2}(4-0)$$
$$= \boxed{-7/2}$$

§5.4 #48 Current $I = \frac{dQ}{dt}$. Then $\int_a^b I(t)\,dt = \int_a^b \frac{dQ}{dt}\,dt = Q(b) - Q(a)$ this is the difference in the charge following in the wire from time $t=a$ to time $t=b$.

§5.4 #52 $f(x)$ is slope of trail in miles from start of trail.

$$f(x) = \frac{dh}{dx} \quad \text{where}$$



$f(x) = \frac{dh}{dx}$

$\underline{dh = f(x)\,dx}$

$$\int_3^5 f(x)\,dx = \int_3^5 \frac{dh}{dx}\,dx$$
$$= \underbrace{h(5) - h(3)}$$

gain in altitude from $x=3$ to $x=5$

§5.4 #55 Find the displacement & distance traveled. Let $v(t) = 3t - 5$ for $0 \le t \le 3$. $v(t) = \frac{dx}{dt}$ thus,

$$\int_0^3 v(t)\,dt = \int_0^3 \frac{dx}{dt}\,dt = \underbrace{x(3) - x(0)}_{\text{displacement}}$$

$$\| $$

$$\int_0^3 (3t-5)\,dt = \left(\frac{3t^2}{2} - 5t\right)\Big|_0^3 = \frac{27}{2} - 15 = \boxed{\frac{-3}{2} = \Delta x}$$

Distance traveled is given by,

$$s = \int_0^3 |v(t)|\,dt = \int_0^3 |3t-5|\,dt = \int_0^{5/3}(5-3t)\,dt + \int_{5/3}^3 (3t-5)\,dt$$

(note $3t - 5 = 0$, $t = 5/3$) shows where to break up |·|.

$$= \left(5t - \frac{3}{2}t^2\right)\Big|_0^{5/3} + \left(\frac{3}{2}t^2 - 5t\right)\Big|_{5/3}^{3}$$
$$= \frac{25}{3} - \frac{3}{2}\left(\frac{25}{9}\right) + \frac{27}{2} - 15 - \frac{3}{2}\left(\frac{25}{9}\right) + \frac{25}{3} = \boxed{\frac{41}{6}}$$

§5.4 # 57) Find velocity at time t and distance traveled in given time interval ②

$a(t) = t + 4 \quad , \quad v(0) = 5 \quad , \quad 0 \leq t \leq 10$

$$a(t) = \frac{dV}{dt} \longrightarrow \underbrace{\int_0^t a(u)\,du}_{} = \int_0^t \frac{dV}{du}\,du = V(t) - V(0)$$

$$\int_0^t (u+4)\,du = \left(\frac{u^2}{2} + 4u\right)\Big|_0^t = \frac{t^2}{2} + 4t$$

$$\therefore \quad \boxed{V(t) = 5 + \frac{t^2}{2} + 4t} \quad \left(\frac{m}{s}\right)$$

Notice $V(t) > 0$ on $0 \leq t \leq 10$ thus $|V(t)| = V(t)$ for $t \in [0,10]$.

$$S = \int_0^{10} |V(t)|\,dt = \int_0^{10} \left(\frac{1}{2}t^2 + 4t + 5\right)dt$$

$$= \left(\frac{1}{6}t^3 + 2t^2 + 5t\right)\Big|_0^{10}$$

$$= \frac{1000}{6} + 200 + 50$$

$$= \frac{1000 + 1200 + 300}{6}$$

$$= \boxed{\frac{2500}{6}} \quad (m)$$

§5.4 # 67)

$$\int (\sin(x) + \sinh(x))\,dx = \boxed{-\cos(x) + \cosh(x) + C}$$

(Note $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ and $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$
thus $\frac{d}{dx}(\cosh(x)) = \frac{d}{dx}\left[\frac{1}{2}(e^x + e^{-x})\right] = \frac{1}{2}(e^x - e^{-x}) = \sinh(x)$.
which shows our antiderivative above is correct)

§5.4#69)

$$\int \left(x^2 + 1 + \frac{1}{x^2+1}\right)dx = \boxed{\frac{1}{3}x^3 + x + \tan^{-1}(x) + C}$$

§5.4#71)

$$\int_0^{1/\sqrt{3}} \frac{t^2-1}{t^4-1}\,dt = \int_0^{1/\sqrt{3}} \frac{\cancel{t^2-1}}{\cancel{(t^2-1)}(t^2+1)}\,dt$$

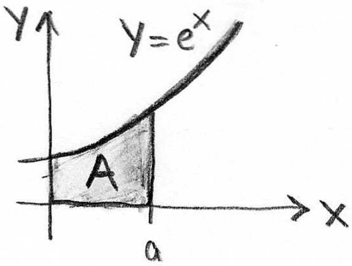
$$= \int_0^{1/\sqrt{3}} \frac{1}{t^2+1}\,dt$$

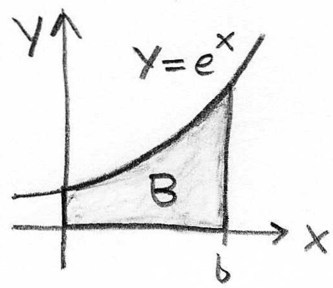
$$= \tan^{-1}(1/\sqrt{3}) - \tan^{-1}(0)$$

$$= \tan^{-1}\left(\frac{1/2}{\sqrt{3}/2}\right), \quad \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ \& notice}$$

$$= \boxed{\pi/6} \qquad \leftarrow \sin(\pi/6) = \frac{1}{2}, \ \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

§5.4#72)





Area B is three times as big as the area A. How are b and a related?

$$\int_0^b e^x\,dx = 3\int_0^a e^x\,dx$$

$$\parallel \qquad\qquad \parallel$$

$$e^b - 1 = 3(e^a - 1)$$

$$e^b = 3e^a - 2$$

$$\boxed{b = \ln(3e^a - 2)}$$

<u>Question</u>: what if we tried this for $f(x) = x$ or x^2 or something besides e^x. Would the answer be different?