Homework 40, Calculus I

§7.8 #5

$$\lim_{x\to 1}\left(\frac{x^2-1}{x^2-x}\right) \underset{(\frac{0}{0})}{\overset{\mathcal{L}}{=}} \lim_{x\to 1}\left(\frac{2x}{2x-1}\right) = \frac{2}{2-1} = \boxed{2}$$

§7.8 #11

$$\lim_{t\to 0}\left(\frac{e^t-1}{t^3}\right) \underset{(\frac{0}{0})}{\overset{\mathcal{L}}{=}} \lim_{t\to 0}\left(\frac{e^t}{3t^2}\right) = \boxed{\infty}$$

§7.8 #13 Assume $q \neq 0$,

$$\lim_{x\to 0}\left(\frac{\tan(px)}{\tan(qx)}\right) \underset{(\frac{0}{0})}{\overset{\mathcal{L}}{=}} \lim_{x\to 0}\left(\frac{p\sec^2(px)}{q\sec^2(qx)}\right) = \frac{p\sec^2(0)}{q\sec^2(0)} = \boxed{\frac{p}{q}}$$

§7.8 #15

$$\lim_{x\to\infty}\left(\frac{\ln(x)}{\sqrt{x}}\right) \underset{(\frac{\infty}{\infty})}{\overset{\mathcal{L}}{=}} \lim_{x\to\infty}\left(\frac{1/x}{1/2\sqrt{x}}\right) = \lim_{x\to\infty}\left(\frac{2\sqrt{x}}{x}\right) = \lim_{x\to\infty}\left(\frac{2}{\sqrt{x}}\right) = \boxed{0}$$

§7.8 #19

$$\lim_{x\to\infty}\left(\frac{e^x}{x^3}\right) \underset{(\frac{\infty}{\infty})}{\overset{\mathcal{L}}{=}} \lim_{x\to\infty}\left(\frac{e^x}{3x^2}\right) \underset{(\frac{\infty}{\infty})}{\overset{\mathcal{L}}{=}} \lim_{x\to\infty}\left(\frac{e^x}{6x}\right) \underset{(\frac{\infty}{\infty})}{\overset{\mathcal{L}}{=}} \lim_{x\to\infty}\left(\frac{e^x}{6}\right) = \boxed{\infty}$$

§7.8 #21

$$\lim_{x\to 0}\left(\frac{e^x-1-x}{x^2}\right) \underset{(\frac{0}{0})}{\overset{\mathcal{L}}{=}} \lim_{x\to 0}\left(\frac{e^x-1}{2x}\right) \underset{(\frac{0}{0})}{\overset{\mathcal{L}}{=}} \lim_{x\to 0}\left(\frac{e^x}{2}\right) = \boxed{\frac{1}{2}}$$

Remark: the notation $\underset{(\frac{0}{0})}{\overset{\mathcal{L}}{=}}$ means I apply L'Hopital's Rule to a limit of type $(0/0)$. The notation $\underset{(\frac{\infty}{\infty})}{\overset{\mathcal{L}}{=}}$ means L'Hopital's Rule is applied to a limit of type $(\frac{\infty}{\infty})$. That is all, we can only apply L'Hopital's Rule to type $(\frac{0}{0})$ or $(\frac{\infty}{\infty})$.

§7.8 #25

$$\lim_{t\to 0}\left(\frac{5^t-3^t}{t}\right) \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{t\to 0}\left(\frac{\ln(5)\,5^t-\ln(3)\,3^t}{1}\right) = \boxed{\ln(5)-\ln(3)}$$

§7.8 #27

$$\lim_{x\to 0}\left(\frac{\sin^{-1}(x)}{x}\right) \underset{(\frac{0}{0})}{\overset{L}{=}} \lim_{x\to 0}\left(\frac{\frac{1}{\sqrt{1-x^2}}}{1}\right) = \lim_{x\to 0}\left(\frac{1}{\sqrt{1-x^2}}\right) = \boxed{1}$$

§7.8 #43

$$\lim_{x\to\infty}\left(x^3e^{-x^2}\right) = \lim_{x\to\infty}\left(\frac{x^3}{e^{x^2}}\right)$$

$$\underset{(\frac{\infty}{\infty})}{\overset{L}{=}} \lim_{x\to\infty}\left(\frac{3x^2}{2xe^{x^2}}\right)$$

$$= \lim_{x\to\infty}\left(\frac{3x}{2e^{x^2}}\right)$$

$$\underset{(\frac{\infty}{\infty})}{\overset{L}{=}} \lim_{x\to\infty}\left(\frac{3}{4xe^{x^2}}\right)$$

$$= \boxed{0}$$

- Exponentials beat polynomials every time.
- What about logs?