In response to another question, when decomposing $\frac{4x^2-1}{(x+3)(x+1)^2}$ "Why not just $\frac{A}{x+1} + \frac{B}{x+1}$?" (for the $\frac{1}{(x+1)^2}$ part.)

Lets try it and see what happens,

$$\frac{4x^2-1}{(x+3)(x+1)^2} = \frac{A}{x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$ (incorrect guess)

$$4x^2-1 = A(x+1)(x+3) + B(x+1)(x+3) + C(x+1)^2$$

Notice then $x=-1 \Rightarrow 3 = A(0) + B(0) + C(0) \Rightarrow \underline{3=0}$

So unless your willing to make $3=0$... lets see how it works if we guess like I advocated in lecture,

$$\frac{4x^2-1}{(x+3)(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$$

$$4x^2-1 = A(x+1)(x+3) + B(x+3) + C(x+1)^2$$

$x=-1$: $3 = B(-1+3) \Rightarrow \boxed{B = 3/2}$

$x=0$: $-1 = 3A + 3B + C = 3A + \frac{9}{2} + C$

$x=-3$: $35 = C(-3+1)^2 = 4C \Rightarrow \boxed{C = \frac{35}{4}}$

Finally solve $A = \frac{-1 - \frac{9}{2} - C}{3} = \frac{\frac{-11}{2} - \frac{35}{4}}{3} = \frac{\frac{-22-35}{4}}{3} = \boxed{\frac{-57}{12} = A}$