7.7 Exercises

1–13 ■ Solve the differential equation.

1. $y'' - 6y' + 8y = 0$
2. $y'' - 4y' + 8y = 0$
3. $y'' + 8y' + 41y = 0$
4. $2y'' - y' - y = 0$
5. $y'' - 2y' + y = 0$
6. $3y'' = 5y'$
7. $4y'' + y = 0$
8. $16y'' + 24y' + 9y = 0$
9. $4y'' + y' = 0$
10. $9y'' + 4y = 0$
11. $\dfrac{d^2y}{dt^2} - 2\dfrac{dy}{dt} - y = 0$
12. $\dfrac{d^2y}{dt^2} - 6\dfrac{dy}{dt} + 4y = 0$
13. $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + y = 0$

14–16 ■ Graph the two basic solutions of the differential equation and several other solutions. What features do the solutions have in common?

14. $6\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = 0$
15. $\dfrac{d^2y}{dx^2} - 8\dfrac{dy}{dx} + 16y = 0$
16. $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 5y = 0$

17–24 ■ Solve the initial-value problem.

17. $2y'' + 5y' + 3y = 0,\quad y(0) = 3,\quad y'(0) = -4$
18. $y'' - 4y = 0,\quad y(0) = 1,\quad y'(0) = 0$
19. $y'' - 2y' + 2y = 0,\quad y(0) = 1,\quad y'(0) = 2$
20. $y'' + 4y' + 6y = 0,\quad y(0) = 2,\quad y'(0) = 4$
21. $y'' - 2y' - 3y = 0,\quad y(1) = 3,\quad y'(1) = 1$
22. $y'' - 2y' + y = 0,\quad y(2) = 0,\quad y'(2) = 1$
23. $y'' + 9y = 0,\quad y(\pi/3) = 0,\quad y'(\pi/3) = 1$
24. $y'' + 4y = 0,\quad y(\pi/6) = 1,\quad y'(\pi/6) = 0$

25–32 ■ Solve the boundary-value problem, if possible.

25. $y'' + 4y' + 4y = 0,\quad y(0) = 0,\quad y(1) = 3$
26. $y'' + 5y' - 6y = 0,\quad y(0) = 0,\quad y(2) = 1$
27. $y'' + y = 0,\quad y(0) = 1,\quad y(\pi) = 0$
28. $y'' + 9y = 0,\quad y(0) = 1,\quad y(\pi/2) = 0$
29. $y'' - y' - 2y = 0,\quad y(-1) = 1,\quad y(1) = 0$
30. $y'' + 4y' + 3y = 0,\quad y(1) = 0,\quad y(3) = 2$
31. $y'' + 4y' + 13y = 0,\quad y(0) = 2,\quad y(\pi/2) = 1$
32. $y'' + 2y' + 5y = 0,\quad y(0) = 1,\quad y(\pi) = 2$

33. (a) Show that the boundary-value problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(L) = 0$ has only the trivial solution $y = 0$ for the cases $\lambda = 0$ and $\lambda < 0$.
    (b) For the case $\lambda > 0$, find the values of λ for which this problem has a nontrivial solution and give the corresponding solution.

34. If a, b, and c are all positive constants and $y(x)$ is a solution of the differential equation $ay'' + by' + cy = 0$, show that $\lim_{x \to \infty} y(x) = 0$.