7.9 Exercises

1. A spring with a 3-kg mass is held stretched 0.6 m beyond its natural length by a force of 20 N. If the spring begins at its equilibrium position but a push gives it an initial velocity of 1.2 m/s, find the position of the mass after t seconds.

2. A spring with a 4-kg mass has natural length 1 m and is maintained stretched to a length of 1.3 m by a force of 24.3 N. If the spring is compressed to a length of 0.8 m and then released with zero velocity, find the position of the mass at any time t.

3. A spring with a mass of 2 kg has damping constant 14, and a force of 6 N is required to keep the spring stretched 0.5 m beyond its natural length. The spring is stretched 1 m beyond its natural length and then released with zero velocity. Find the position of the mass at any time t.

4. A spring with a mass of 3 kg has damping constant 30 and spring constant 123.
(a) Find the position of the mass at time t if it starts at the equilibrium position with a velocity of 2 m/s.
(b) Graph the position function of the mass.

5. For the spring in Exercise 3, find the mass that would produce critical damping.

6. For the spring in Exercise 4, find the damping constant that would produce critical damping.

7. A spring has a mass of 1 kg and its spring constant is $k = 100$. The spring is released at a point 0.1 m above its equilibrium position. Graph the position function for the following values of the damping constant c: 10, 15, 20, 25, 30. What type of damping occurs in each case?

8. A spring has a mass of 1 kg and its damping constant is $c = 10$. The spring starts from its equilibrium position with a velocity of 1 m/s. Graph the position function for the following values of the spring constant k: 10, 20, 25, 30, 40. What type of damping occurs in each case?

9. Suppose a spring has mass m and spring constant k and let $\omega = \sqrt{k/m}$. Suppose that the damping constant is so small that the damping force is negligible. If an external force $F(t) = F_0 \cos \omega_0 t$ is applied, where $\omega_0 \neq \omega$, use the method of undetermined coefficients to show that the motion of the mass is described by Equation 6.

10. As in Exercise 9, consider a spring with mass m, spring constant k, and damping constant $c = 0$, and let $\omega = \sqrt{k/m}$. If an external force $F(t) = F_0 \cos \omega t$ is applied (the applied frequency equals the natural frequency), use the method of undetermined coefficients to show that the motion of the mass is given by $x(t) = c_1 \cos \omega t + c_2 \sin \omega t + (F_0/(2m\omega))t \sin \omega t$.

11. A series circuit consists of a resistor with $R = 20\ \Omega$, an inductor with $L = 1$ H, a capacitor with $C = 0.002$ F, and a 12-V battery. If the initial charge and current are both 0, find the charge and current at time t.

12. A series circuit contains a resistor with $R = 24\ \Omega$, an inductor with $L = 2$ H, a capacitor with $C = 0.005$ F, and a 12-V battery. The initial charge is $Q = 0.001$ C and the initial current is 0.
(a) Find the charge and current at time t.
(b) Graph the charge and current functions.

13. The battery in Exercise 11 is replaced by a generator producing a voltage of $E(t) = 12 \sin 10t$. Find the charge at time t.

14. The battery in Exercise 12 is replaced by a generator producing a voltage of $E(t) = 12 \sin 10t$.
(a) Find the charge at time t.
(b) Graph the charge function.

15. Verify that the solution to Equation 1 can be written in the form $x(t) = A \cos(\omega t + \delta)$.

16. The figure shows a pendulum with length L and the angle θ from the vertical to the pendulum. It can be shown that θ, as a function of time, satisfies the nonlinear differential equation

$$\frac{d^2\theta}{dt^2} + \frac{g}{L} \sin \theta = 0$$

where g is the acceleration due to gravity. For small values of θ we can use the linear approximation $\sin \theta \approx \theta$ and then the differential equation becomes linear.
(a) Find the equation of motion of a pendulum with length 1 m if θ is initially 0.2 rad and the initial angular velocity is $d\theta/dt = 1$ rad/s.
(b) What is the maximum angle from the vertical?
(c) What is the period of the pendulum (that is, the time to complete one back-and-forth swing)?
(d) When will the pendulum first be vertical?
(e) What is the angular velocity when the pendulum is vertical?