MA241-006: Calculus II

Instructor: Mr. James Cook
Test: #1 Form B
Date: Wednesday, Febuary 1, 2006

Directions: You must show **ALL** your work to receive credit for problems 1 and 2.

1. (55 pts) Integrate the following functions:

(a)(10pts)

$$\int 2xe^{x^2}\,dx = \int e^u\,du = e^u + C = \boxed{e^{x^2} + C}$$

$$\left(\text{let } \begin{array}{l} u = x^2 \\ du = 2x\,dx \end{array}\right)$$

(b)(10pts)

$$\int \underbrace{\ln(x)}_{u}\,\underbrace{dx}_{dv} = \ln(x)\cdot x - \int x\,\frac{dx}{x} = \boxed{x\ln(x) - x + C}$$

$u = \ln(x)$
$du = \left(\frac{1}{x}\right)dx$

(c)(5pts) You may recall that if we let $u = \sec(\theta) + \tan(\theta)$ then it can be shown (you do not have to perform the required differentiations exc...) that $\sec(\theta)\, d\theta = \frac{du}{u}$.

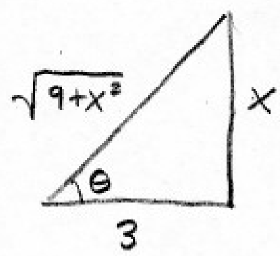
$$
\begin{aligned}
\int \sec(\theta)\, d\theta &= \int \frac{du}{u} \\
&= \ln|u| + C \\
&= \boxed{\ln|\sec\theta + \tan\theta| + C}
\end{aligned}
$$

(d)(15pts) You should find part c useful in part of this computation.

$$
\begin{aligned}
\int \frac{1}{\sqrt{9+x^2}}\, dx &= \int \frac{1}{\sqrt{9\sec^2\theta}}\, 3\sec^2\theta\, d\theta \\
&= \int \sec\theta\, d\theta \\
&= \ln|\sec\theta + \tan\theta| + C \\
&= \boxed{\ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right| + C}
\end{aligned}
$$

$$
\left(
\begin{aligned}
x &= 3\tan\theta \\
9 + x^2 &= 9 + 9\tan^2\theta = 9\sec^2\theta \\
dx &= 3\sec^2\theta\, d\theta
\end{aligned}
\right)
$$

(using 1c.)



$\tan\theta = \frac{x}{3}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{9+x^2}}{3}$

(e)(15pts) Completing the square in the denominator should help.

$$\int \frac{x}{x^2+4x+5}\,dx = \int \frac{x}{(x+2)^2+1}\,dx$$

$$= \int \frac{u-2}{u^2+1}\,du \qquad \begin{pmatrix} u = x+2 \\ x = u-2 \\ dx = du \end{pmatrix}$$

$$= \int \frac{u\,du}{u^2+1} - 2\int \frac{du}{u^2+1}$$

$$= \int \frac{\frac{1}{2}dw}{w} - 2\tan^{-1}(u) \qquad \begin{pmatrix} w = u^2+1 \\ dw = 2u\,du \\ \frac{dw}{2} = u\,du \end{pmatrix}$$

$$= \frac{1}{2}\ln|w| - 2\tan^{-1}(u) + C$$

$$= \frac{1}{2}\ln|u^2+1| - 2\tan^{-1}(u) + C$$

$$= \boxed{\frac{1}{2}\ln|x^2+4x+5| - 2\tan^{-1}(x+2) + C}$$

2. (10pts) Compute the improper integral. Write the limit involving ∞ explicitly.

$$\int_0^\infty e^{-x}\,dx = \lim_{t\to\infty}\int_0^t e^{-x}\,dx$$

note $\frac{d}{dx}(-e^{-x}) = e^{-x}$

there fore

$\int e^{-x}dx = -e^{-x} + C$

then use F.T.C.

$$= \lim_{t\to\infty}\left(-e^{-t} + e^{0}\right)$$

$$= \lim_{t\to\infty}\left(\underbrace{\frac{-1}{e^{t}}}_{0} + 1\right)$$

$$= \boxed{1}$$

No work is required for problems 3-8, just follow the instructions and circle the appropriate answer. If more than one answer is selected you will receive no credit.

3. (10 pts) Given the rational function below

$$f(x) = \frac{x+10}{x^2(x+1)(x^2-4)(x^2+5)^2}$$

Circle the correct partial fractions decomposition of f(x) below (assume that A,B,C,... are undetermined constants),

(a)

$$f(x) = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{x+2} + \frac{E}{x-2} + \frac{Fx+G}{(x^2+5)^2} + \frac{Hx+I}{x^2+5}$$

(b)

$$f(x) = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{Dx+E}{x^2-4} + \frac{Fx+G}{(x^2+5)^2} + \frac{Hx+I}{x^2+5}$$

(c)

$$f(x) = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{Dx+E}{x^2-4} + \frac{Fx^3+Gx^2}{(x^2+5)^2} + \frac{Hx+I}{x^2+5}$$

(d)

$$f(x) = \frac{Ax+B}{x^2} + \frac{C}{x+1} + \frac{Dx+E}{x^2-4} + \frac{Fx+G}{(x^2+5)^2} + \frac{Hx+I}{x^2+5}$$

4. (5 pts) Circle the correct trigonometric substitution to remove the squareroot from $\sqrt{9-x^2}$.

(a) $x = 3\sec(\theta)$
(b) $x = 3\cos(\theta)$
(c) $\theta = 3\cos(x)$
(d) $x = 3\tan(\theta)$

$$\sqrt{9-x^2} = \sqrt{9-(3\cos\theta)^2} = \sqrt{9(1-\cos^2\theta)} = \sqrt{9\sin^2\theta}$$
$$= \boxed{3\sin\theta}$$

5. (5 pts) **True** or **False** (circle one)

$$\int f(x)g(x)dx = \int f(x)dx \int g(x)dx$$

Counterexample: $\int x\,x\,dx = \int x^2 dx = \frac{1}{3}x^3 + C$

$\left(\int x dx\right)\left(\int x dx\right) = \left(\frac{1}{2}x^2 + c\right)\left(\frac{1}{2}x^2 + c\right)$ not equal.

6. (5 pts) **True** or **False** (circle one)

$$\int \frac{1}{f(x)}\,dx = \ln|f(x)| + C$$

where C is a constant and $f(x)$ is any nonzero continuous function.

$$\int \frac{1}{x^2}\,dx = \frac{-1}{x} + c \neq \ln|x^2| + c$$

Only works when $f(x) = x$. (As far as I can think of)

7. (5 pts) It is known that the function $f(x) = \frac{sin(x)}{x}$ has second and fourth derivatives whose absolute values are bounded by the same number K on (1,2). You forgot to bring your calculator on vacation but your grandma wants to know what the $\int_1^2 \frac{sin(x)}{x} dx$ is to within an error of 0.001. Circle the method you should use to minimize grandma's waiting time.

(a) Dwight's rule
(b) Agent Michael Scott's Rule
(c) Trapezoid rule
(d) Midpoint rule
(e) Simpson's rule

(Incidentally, your grandma still has her trigonometry text from highschool, complete with values of sin(x) correct to 0.0001 so finding values of f(x) poses no real difficulty.)

8. (5 pts) Circle the correct trigonometric identity below.

(a) $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

(b) $\sin^2(x) = \frac{1}{2}(1 + \cos(2x))$