$\boxed{8.1 \ \#13}$

$$\lim_{n\to\infty} \frac{(n+2)!}{n!} = \lim_{n\to\infty} \frac{(n+2)(n+1)\cancel{(n)}\cancel{(n-1)}\cdots\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}$$

$$= \lim_{n\to\infty} (n+2)(n+1)$$

$$\boxed{=\infty}$$

$\boxed{8.1 \ \#11}$

$$\lim_{n\to\infty}\left(\frac{2^n}{3^{n+1}}\right) = \lim_{n\to\infty}\left(\frac{1}{3}\frac{2^n}{3^n}\right) = \frac{1}{3}\lim_{n\to\infty}\left(\frac{2}{3}\right)^n = \frac{1}{3}(0) = \boxed{0}$$

converges

~~[illegible]~~ $\boxed{8.1 \ \#9}$

$$\lim_{n\to\infty}\left(\frac{3+5n^2}{n+n^2}\right) = \lim_{n\to\infty}\left(\frac{3/n^2+5}{1/n+1}\right) = \boxed{5}$$

0

$\boxed{8.2 \ \#13}$

$$\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n-1} = \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} = \frac{5}{1-2/3} = \frac{5}{1/3} = \boxed{15}$$

$|r|<1$

$\boxed{8.2 \ \#17}$

$$\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2}\left(\sum_{n=1}^{\infty}\frac{1}{n}\right) = \boxed{\infty}$$

$$\int_1^\infty \frac{1}{2x}\,dx = \frac{1}{2}\lim_{t\to\infty}\left(\ln(t) - \ln(1)\right) = \infty \Rightarrow \text{by } \int\text{-test}$$

it diverges

8.2 #19 $\quad$ *first test to think about*

$\sum_{k=2}^{\infty} \frac{k^2}{k^2-1}$ diverges by the k^{th} term test $\quad \lim_{k\to\infty}\left(\frac{k^2}{k^2-1}\right) = 1 \neq 0$

diverges

8.2 #21

$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n \quad = \left(\frac{1}{3}+\frac{1}{9}\ldots\right)+\left(\frac{2}{3}+\frac{4}{9}+\frac{8}{27}\ldots\right)$

$= \left(\frac{1/3}{1-1/3}\right) + \left(\frac{2/3}{1-2/3}\right) = \boxed{5/2}$ converges

8.2 #23

$\sum_{n=1}^{\infty} \sqrt[n]{2} = \quad \lim_{n\to\infty}\left(\sqrt[n]{2}\right) = y$

$\lim_{n\to\infty} e^{\ln \sqrt[n]{2}} = \lim_{n\to\infty} e^{\frac{1}{n}\ln(2)} = e^{\lim \frac{1}{n}\ln(2) \to 0}$

b/c e is cont.

$e^0 = 1 \neq$ therefore diverges

8.2 #25 $\quad \sum_{n=1}^{\infty} \tan^{-1}(n) \quad a_n = \tan^{-1}(n) \to \frac{\pi}{2} \neq 0$ as $n\to\infty$

diverges by n^{th} term test

8.2 #31 $\quad .\overline{222}$

$\frac{2}{10}+\frac{2}{100}\ldots = S \quad a = \frac{2}{10} \quad r = \frac{1}{10} = \frac{2}{100}/\frac{2}{10}$

$= \frac{a}{1-r} = \frac{2/10}{1-1/10} = \boxed{\frac{2}{9}}$

$\boxed{8.3 \ \#7}$

$\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges by p-series test
$p = 4 > 1$

int test: $\int_1^{\infty} \frac{1}{x^4}\,dx = \lim_{t\to\infty}\left(\frac{-1}{3t^3} + \frac{1}{3}\right) = \boxed{\frac{1}{3}}$

0

notice x^4 is cont. pos. and $\frac{d}{dx}\left(\frac{1}{x^4}\right) = \frac{-4}{x^5} < 0$ on $x \geq 1$
its decreasing, thus $\int$-test $\sum \frac{1}{n^4}$ converges

look for test → $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$ note $f(x) = \frac{1}{1+x^2}$ has $f(n) = \frac{1}{1+n^2}$

$f(x)$ is continuous, positive, $f'(x) = \frac{-2x}{(1+x^2)^2} < 0$ on $x \geq 1$ $\therefore$ f is decreasing

$\int_1^{\infty} \frac{1}{1+x^2} = \lim_{t\to\infty}\left(\tan^{-1}(t) - \tan^{-1}(1)\right) = \frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$ converges

$\therefore$ by $\int$-test $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges

$\frac{dx}{x\ln(x)} = \int_4^{\infty} \frac{du}{u}$ $\quad u = \ln(x)$ $\quad = \ln(u) + C$
$= \ln|\ln(x)| + C$

$\sum_{n=2}^{\infty} \frac{1}{n\ln(x)}$ $\quad f(x) = \frac{1}{x\ln(x)}$ continuous on $x \geq 2$ and positive

$f'(x) = \frac{-1}{(x\ln(x))^2}\left[\ln(x) + \frac{x}{x}\right] < 0$ on $x \geq 2$ $\therefore$ f is decreasing

$\int_2^{\infty} \frac{dx}{x\ln(x)} = \lim_{t\to\infty}\left[\ln|\ln(t)| - \ln|\ln(t)|\right] = \infty$ $\therefore$ $\sum_{n=2}^{\infty} \frac{1}{\ln(\ln(x))}$ " $\boxed{\text{diverges}}$

$\boxed{\infty}$