YOUR NAME HERE: ____

MA 341, Introduction to Differential Equations

Instructor: James Cook
Practice Test : n-th order ODEs
Date: Tuesday, September 18, 2007

Directions: Show your work, if you doubt that you've shown enough detail then ask. If you need additional paper please ask. There are 106 pts to be earned, 6pts are bonus.

WARNING: this is a practice test, I will make the actual test shorter. Some problems will remain essentially the same but there is obviously too much for 75 minutes here.

1. (5pts) Assume that k is a given but unknown constant. Solve

$$\frac{dT}{dt} = k(T - 100)$$

your answer will include the constant k and some constant of integration.

2. (5 pts) Suppose a cup of coffee cools according to Newton's Law of Cooling. Further suppose that at $t = 0$ the coffee is at $T = 160$ then after one minute the coffee cools to $T = 140$. If the room the coffee is cooling in has an ambient temperature of 100 then find the temperature at time t. If my daughter Hannah likes to steal my coffee once it cools to 90 degrees do I need to worry about her sneaky plans to sip my coffee when I'm not looking ?

3. (10pts) Given that $v(0) = 10$ solve

$$\frac{dv}{dt} = -v^2$$

If v represents the velocity of a cat thrown horizontally with an initial velocity of 10 then does the cat ever come to rest ? Here you can interpret the term $-v^2$ as a frictional force. For a bonus point find if the position of the cat is bounded, use $v = dx/dt$.

4. (10pts) Solve

$$y + \frac{x}{2}\frac{dy}{dx} = \frac{1}{2x^3}$$

5. (10 pts) Find the solution of

$$e^x(y - x)dx + (1 + e^x)dy = 0$$

that passes through the point $(0, 0)$. Notice this is an exact equation.

6. (20 pts) Find the general solutions to the following differential equations, as usual we denote $D = d/dx$ and $y' = dy/dx$ etc...
(a.) $y'' + 5y' + 6y = 0$
(b.) $y'' + y' + y = 0$
(c.) $(D^3 - 5D^2 + 6D)[y] = 0$
(d.) $(D^2 + D + 1)^2[y] = 0$

7. (20 pts) Find the general solution of

$$y'' + 4y = x^2 + 5e^x$$

8. (15 pts) Find the general solution of

$$y'' + 5y' + 6y = cos(x) + 2sin(x)$$

9. (15 pts) Find the general solution of

$$y'' + y = 3x^2 + x + 1$$

10. (10 pts) Find the particular solution of

$$y'' + 4y = \tan(2x)$$

via the method of variation of parameters. Recall,

$$v_1 = \int \frac{-gy_2}{y_1\frac{dy_2}{dx} - y_2\frac{dy_1}{dx}}\, dx \qquad v_2 = \int \frac{gy_1}{y_1\frac{dy_2}{dx} - y_2\frac{dy_1}{dx}}\, dx$$

11. (6 pts) Find the general solution of

$$y'' + 4y = x^2 + 5e^x + \tan(2x)$$

please use the previous two problems for the sake of time.

12. (10 pts) Use the method of anihilators to find the correct form for the particular solution (don't find $A, B, C, ...$ just set it up) of the following differential equation, $D = d/dx$ as usual,

$$(D + 1)(D^2 + 1)(D - 3)^2[y](x) = e^{-x}$$

do the same for

$$(D + 1)(D^2 + 1)(D - 3)^2[y](x) = cos(x) + e^{-x}$$

and

$$(D + 1)(D^2 + 1)(D - 3)^2[y](x) = xcos(x) + sin(x)$$

13. (10pts) Use $e^{ix} = cos(x) + isin(x)$ to show that

$$sin^2(x) = \frac{1}{2}(1 - cos(2x))$$

it is useful to first recall the formula for cos(x) and sin(x) in terms of e^{ix}.