FINAL EXAM GUIDE : MA 430

- COVERS : CHAPTERS 9 – 12
- HOMEWORKS MOST RELEVANT : 28 – 36, 39 – 40, 42 – 44, 49 – 53.
- DEFINITIONS YOU SHOULD KNOW (MEMORIZE)
  - multilinear map on V, V^*, $V \times V$, $V^* \times V$, etc...
  - tensor products $e_i \otimes e_j$ and $e^i \otimes e^j$ etc...
  - symmetric and antisymmetric tensors (
  - wedge product abstract & concrete tensor product version.
  - determinant of matrix via wedge product
  - exterior derivative & integrals of differential forms
  - linear independence & spanning, basis set
- I will give you the def^{n} of Hodge duality, but I assume you know how to raise/lower indices
- Be able to prove things in homework and,
  - If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(A) = ad - bc$.
  - Linearly dependent vectors $\{v_1, v_2, \dots, v_p\}$ wedge to zero, $v_1 \wedge v_2 \wedge \dots \wedge v_p = 0$
  - GIVEN THE TABLE, show $*\Phi_A = \omega_A$, $*\omega_B = \Phi_B$ and $\omega_A \wedge \omega_B = \Phi_{A \times B}$
  - Show that for a function f and vector field F
    $df = \omega_{\nabla f}$, $d\Phi_F = (\nabla \cdot F)\, dx \wedge dy \wedge dz$, $d\omega_F = \Phi_{\nabla \times F}$
  - Show for any p-form α that $d(d\alpha) = 0$
  - Given that $\int_M d\alpha = \int_{\partial M} \alpha$ show that the ordinary Stoke's & Gauss' Th^{m}'s follow, I'll remind you that $\int_S (\nabla \times F) \cdot dA = \int_{\partial S} F \cdot dl$ & $\int_V (\nabla \cdot G)\, d\tau = \int_{\partial V} G \cdot dA$
  - If $vol(M) = \int_M dx_1 \wedge dx_2 \wedge \dots \wedge dx_n$ show that $vol(M) = \int_{\partial M} x_1\, dx_2 \wedge dx_3 \wedge \dots \wedge dx_n$.
  - GIVEN THE TABLE, if $\mathcal{J} = -\rho dt + \omega_J$ find $*\mathcal{J}$.
  - GIVEN that $F = dA$ show $F = -\omega_E \wedge dt + \Phi_B$, I also give you $A = -V dt + \omega_{\vec{A}}$ and how E & B relate to E, B namely $E = -\nabla V - \frac{\partial \vec{A}}{\partial t}$, $B = \nabla \times \vec{A}$

- Be able to prove things in homework and,
  - Given $F = -\omega_E \wedge dt + \Phi_B$ and Table of Hodge dualities show that $*F = -\omega_B \wedge dt + \Phi_E$
  - Given $F = -\omega_E \wedge dt + \Phi_B$, $*F = -\omega_B \wedge dt + \Phi_E$ and $*\mathcal{J}$ show that Maxwell's Eq^n's (I'll quote them on test) are implicit within $d(*F) = *\mathcal{J}$ and $dF = 0$.

<u>Remark</u>: You should know how to take Hodge dual in Euclidean or Minkowski Space. As we discussed today the method we used previously was slightly flawed but I allow you to still use it. The correct arguments will appear in the corrected notes soon. I also plan to add another chapter collecting the various digressions and elaborations.