1. SHOW THAT $(\vec{A}\times\vec{B})\cdot\vec{B}=0$ USING THE REPEATED INDEX NOTATION. A & B ARE VECTORS IN $\mathbb{R}^3$.

$(\vec{A}\times\vec{B})\cdot\vec{B} = (\vec{A}\times\vec{B})_k B_k$

$= \epsilon_{ijk} A_i B_j B_k$ * Substitute $\epsilon_{ijk}A_iB_j$ for $(\vec{A}\times\vec{B})_k$

$= \epsilon_{imp} A_i B_m B_p$ * Substitute m & p for j & k

$= \epsilon_{imp} A_i B_p B_m$ * $B_mB_p = B_pB_m \rightarrow$ symmetric

$= -\epsilon_{ipm} A_i B_p B_m$ * $\epsilon_{imp} = -\epsilon_{ipm} \rightarrow$ antisymmetric

$= -(\vec{A}\times\vec{B})_m B_m$ * substitute $(\vec{A}\times\vec{B})_m$ for $\epsilon_{ipm}A_iB_p$

$(\vec{A}\times\vec{B})\cdot\vec{B} = -(\vec{A}\times\vec{B})\cdot\vec{B}$

$+(\vec{A}\times\vec{B})\cdot\vec{B} \quad +(\vec{A}\times\vec{B})\cdot\vec{B}$

$2(\vec{A}\times\vec{B})\cdot\vec{B} = 0$

$(\vec{A}\times\vec{B})\cdot\vec{B} = 0$

✓

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2. VERIFY THE IDENTITY $\epsilon_{ijk}\epsilon_{mjk} = 2\delta_{im}$ IN THE PARTICULAR CASES:

(a) $i=1,\ m=1$

$\epsilon_{ijk}\epsilon_{ijk} = 2\delta_{11} = 2\cdot 1 = 2$ * $\delta_{11}=1$ SINCE $i=m$

* NEED TO SHOW $\epsilon_{ijk}\epsilon_{ijk} = 2$

$\rightarrow \epsilon_{111}\epsilon_{111} + \epsilon_{112}\epsilon_{112} + \epsilon_{113}\epsilon_{113} + \epsilon_{121}\epsilon_{121} + \epsilon_{122}\epsilon_{122} + \epsilon_{123}\epsilon_{123} + \epsilon_{131}\epsilon_{131} + \epsilon_{132}\epsilon_{132} + \epsilon_{133}\epsilon_{133}$

$= 0\cdot0 + 0\cdot0 + 0\cdot0 + 0\cdot0 + 0\cdot0 + 1\cdot1 + 0\cdot0 + (-1)\cdot(-1) + 0\cdot0$

$= 1 + 1$

$= \boxed{2}$ ✓

(b) $i=1, m=2$

$\epsilon_{1jk}\epsilon_{2jk} = 2\delta_{12} = 2\cdot 0 = 0$ $\quad * \delta_{12} = 0$ SINCE $i \neq m$

* NEED TO SHOW $\epsilon_{1jk}\epsilon_{2jk} = 0$

$\rightarrow \epsilon_{111}\epsilon_{211} + \epsilon_{112}\epsilon_{212} + \epsilon_{113}\epsilon_{213} + \epsilon_{121}\epsilon_{221} + \epsilon_{122}\epsilon_{222} + \epsilon_{123}\epsilon_{223} + \epsilon_{131}\epsilon_{231} + \epsilon_{132}\epsilon_{232} + \epsilon_{133}\epsilon_{233}$

$= 0\cdot 0 + 0\cdot 0 + 0\cdot(-1) + 0\cdot 0 + 0\cdot 0 + 1\cdot 0 + 0\cdot 1 + (-1)\cdot 0 + 0\cdot 0$

$= \boxed{0}$

(c) $m=1, i=3$

$\epsilon_{3jk}\epsilon_{1jk} = 2\delta_{31} = 2\cdot 0 = 0$ $\quad * \delta_{31} = 0$ SINCE $i \neq m$

* NEED TO SHOW $\epsilon_{3jk}\epsilon_{1jk} = 0$

$\rightarrow \epsilon_{311}\epsilon_{111} + \epsilon_{312}\epsilon_{112} + \epsilon_{313}\epsilon_{113} + \epsilon_{321}\epsilon_{121} + \epsilon_{322}\epsilon_{122} + \epsilon_{323}\epsilon_{123} + \epsilon_{331}\epsilon_{131} + \epsilon_{332}\epsilon_{132} + \epsilon_{333}\epsilon_{133}$

$= 0\cdot 0 + 1\cdot 0 + 0\cdot 0 + (-1)\cdot 0 + 0\cdot 0 + 0\cdot 1 + 0\cdot 0 + 0\cdot(-1) + 0\cdot 0$

$= \boxed{0}$

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3. VERIFY THE IDENTITY $\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{jl}\delta_{im}$ IN THE PARTICULAR CASES:

(a) $i=1, j=2, l=1, m=1$

$\epsilon_{12k}\epsilon_{k11} = \delta_{11}\delta_{21} - \delta_{21}\delta_{11} = 1\cdot 0 - 0\cdot 1 = 0$

* NEED TO SHOW $\epsilon_{12k}\epsilon_{k11} = 0$

$\rightarrow \epsilon_{121}\epsilon_{111} + \epsilon_{122}\epsilon_{112} + \epsilon_{123}\epsilon_{113}$

$= 0\cdot 0 + 0\cdot 0 + 1\cdot 0$

$= \boxed{0}$

(b) $i=1,\ j=2,\ l=1,\ m=2$

$\epsilon_{12k}\epsilon_{k12} = \delta_{11}\delta_{22} - \delta_{21}\delta_{12} = 1\cdot 1 - 0\cdot 0 = 1$

$*$ NEED TO SHOW $\epsilon_{12k}\epsilon_{k12} = 1$

$\rightarrow \epsilon_{121}\epsilon_{112} + \epsilon_{122}\epsilon_{212} + \epsilon_{123}\epsilon_{312}$

$= 0\cdot 0 + 0\cdot 0 + 1\cdot 1$

$= \boxed{1}$

(c) $i=1,\ j=2,\ l=2,\ m=3$

$\epsilon_{12k}\epsilon_{k23} = \delta_{12}\delta_{23} - \delta_{22}\delta_{13} = 0\cdot 0 - 1\cdot 0 = 0$

$*$ NEED TO SHOW $\epsilon_{12k}\epsilon_{k23} = 0$

$\rightarrow \epsilon_{121}\epsilon_{123} + \epsilon_{122}\epsilon_{223} + \epsilon_{123}\epsilon_{323}$

$= 0\cdot 1 + 0\cdot 0 + 1\cdot 0$

$= \boxed{0}$

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4. USE THE IDENTITIES IN PROBLEMS 2 & 3 PLUS THE ANTISYMMETRY OF ϵ_{ijk} TO FIND NICE FORMULAS FOR THE FOLLOWING IN TERMS OF δ'S OR CONSTANTS.

$\epsilon_{kij}\epsilon_{mjk} = \epsilon_{ijk}\epsilon_{mjk} = 2\delta_{im}.$

(a) $\epsilon_{kij}\epsilon_{mjk} \rightarrow \epsilon_{kij} = \epsilon_{ijk} \Rightarrow \epsilon_{ijk}\epsilon_{mjk} = \boxed{2\delta_{im}}$

~~well~~ $\cancel{\epsilon_{kij}\epsilon_{mjk} = \epsilon_{ijk}\epsilon_{kmj}}$.

(b) $\epsilon_{abc}\epsilon_{cef} \Rightarrow \boxed{\delta_{ae}\delta_{bf} - \delta_{be}\delta_{af}}$ ✓

$\epsilon_{abc}\epsilon_{cef} = \delta_{ae}\delta_{bf} - \delta_{be}\delta_{af}.$

(c) $\epsilon_{ijk}\epsilon_{ijk} \Rightarrow 2\delta_{ii} = 2\cdot 3 = \boxed{6}$

$\epsilon_{ijk}\epsilon_{ijk} = 2\delta_{ii} = 2(3) = 6.$

(d) $\epsilon_{kij}\epsilon_{lmk} \rightarrow \epsilon_{kij} = \epsilon_{ijk}$ & $\epsilon_{lmk} = \epsilon_{klm} \Rightarrow \epsilon_{ijk}\epsilon_{klm} = \boxed{\delta_{il}\delta_{jm} - \delta_{jl}\delta_{im}}$

$\epsilon_{kij}\epsilon_{lmk}$ "

("$\Rightarrow$" ambiguous; $\equiv$ better)

7. WORK ENERGY THEOREM. LET $K_i = \frac{1}{2}mv_i^2$ AND LET $F = m\frac{dv}{dt}$ WHERE m IS A CONSTANT. SHOW THAT $K_f - K_i = \int_{x(t_i)}^{x(t_f)} F(x)dx$. (HINT: $v = \frac{dx}{dt}$, $\frac{dv}{dt} = \frac{dx}{dt}\frac{dv}{dx}$)

• $K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

* NEED TO SHOW $\int_{x(t_i)}^{x(t_f)} F(x)dx = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$\int_{x(t_i)}^{x(t_f)} F(x)dx$

$\int_{x(t_i)}^{x(t_f)} m\frac{dv}{dt}dx$ * Substitute $m\frac{dv}{dt}$ for F

$\int_{x(t_i)}^{x(t_f)} m\frac{dx}{dt}\frac{dv}{dx}dx$ * Substitute $\frac{dx}{dt}\frac{dv}{dx}$ for $\frac{dv}{dt}$

$\int_{x(t_i)}^{x(t_f)} mv\frac{dv}{dx}dx$ * Substitute v for $\frac{dx}{dt}$

$\int_{v_i}^{v_f} mv\,dv$ * Cancel THE TWO dx TERMS & CHANGE INTEGRATION ENDPTS. SINCE NOW IN TERMS OF v.

$m\int_{v_i}^{v_f} v\,dv$ * SINCE m IS CONSTANT, MOVE IT IN FRONT OF INTEGRAL

$m\left(\frac{1}{2}v^2\right)\Big|_{v_i}^{v_f}$ * INTEGRATE v

$m\left(\frac{1}{2}v_f^2 - \frac{1}{2}v_i^2\right)$ * DISTRIBUTE m

$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$\therefore K_f - K_i = \int_{x(t_i)}^{x(t_f)} F(x)dx.$ ✓