Math 231 Homework Project II: Derivatives:

Follow instructions. Be careful to answer all the questions raised in each part. Please turn in neat work with problems clearly labeled and your name on each page. Thanks. Some of these problems are hard. You are not alone. If you get stuck, phone a friend. Or email me, or stop by office hours. Start early though, otherwise we may not be able to resolve your questions in time. Also, come to class. I may inadvertently work part of the homework. I plan for there to be 30 total problems. This assignment has 9 problems, this means it is worth 9pts of your final grade.

PROBLEM 12: Let $f(x,y) = ye^{-xy}$. Find the direction(s) in which the directional derivative of f at the point (0,2) has the value 1. Give the direction(or directions) in terms of unit vector(s).

Reminder: the parametrization of a surface is a mapping X from $U \subseteq \mathbb{R}^2$ to $\mathbb{R}^3$. For full credit you must supply both the equations of $X(u,v) =< X_1(u,v), X_2(u,v), X_3(u,v) >$ and the domain U from which the point (u,v) are taken. Of course the parameters need not be labeled by "u" and "v". You can use any two appropriate chosen variables.

PROBLEM 13: Find parametrizations for the surfaces described in problems 19, 20 and 21 of section 17.6. Find the tangent plane at (1,2,-3) for #19. Find the tangent plane at (0,0,1) for #20. Find the tangent plane at (0,1,0) for #21. Then plot both the surface and tangent plane for each of the problems using Mathematica. (this should produce three separate graphs to print out)

PROBLEM 14: Find parametrizations for the surfaces described in problems 22, 23 and 24 of section 17.6. Find the tangent plane at $(0,\sqrt{2},1)$ for #22. Find the tangent plane at (0,0,2) for #23. Find the tangent plane at (4,0,0) for #24. Then plot both the surface and tangent plane for each of the problems using Mathematica. (this should produce three separate graphs to print out)

PROBLEM 15: Find parametrizations for the surfaces described in problems 25 and 26 of section 17.6. Find the tangent plane at (3,4,0) for #25. Find the tangent plane at (0,1,3) for #26. Find the tangent plane at (4,0,0) for #24. Then plot both the surface and tangent plane for each of the problems using Mathematica. (this should produce two separate graphs to print out, what is the difference between the tangent plane and the surface for #26? There is a difference.)

PROBLEM 16: Calculate the Jacobian matrices for the various functions discussed in E70 on page 310 of my notes. Work out how the chain rule for the general derivative gives back the chain rules given before in section 15.5. In other words, work out my example so you can see how the Jacobian matrix reproduces the chain rules given previously.

PROBLEM 17: Solve problems 13 and 29 of section 15.7. Back up any claims via appropriate calculus and theorems from my notes or the text.

PROBLEM 18: Let $s = \sqrt{x^2+y^2}$ and suppose $\beta = \tan^{-1}(\frac{y}{x})$. Calculate ∇s and $\nabla \beta$. Then calculate unit vectors in the same direction (at an arbitrary point), that is calculate

$$\widehat{u}_s = \frac{1}{|\nabla s|}\nabla s \qquad \widehat{u}_\beta = \frac{1}{|\nabla \beta|}\nabla \beta.$$

PROBLEM 19: Show that if $f : \mathbb{R}^2 \to \mathbb{R}^2$ is differentiable everywhere then $\nabla f =< f_x, f_y >$ can be rewritten as

$$\nabla f = \frac{\partial f}{\partial s}\widehat{u}_s + \frac{1}{s}\frac{\partial f}{\partial \beta}\widehat{u}_\beta.$$

Here you should use the result of problem 18. The solution will require the use of the chain rule for several variables and some vector algebra.

PROBLEM 20: Solve problems 8, 12, 16 and 21 of section 16.3. Motivate any change of integration order with appropriate graphs.