Math 231 Homework Project III: Vector Calculus:

Follow instructions. Be careful to answer all the questions raised in each part. Please turn in neat work with problems clearly labeled and your name on each page. Thanks. Some of these problems are hard. You are not alone. If you get stuck, phone a friend. Or email me, or stop by office hours. Start early though, otherwise we may not be able to resolve your questions in time. I plan for there to be 30 total problems. This assignment has 10 problems, this means it is worth 10pts of your final grade.

Remark: *Cartesian coordinates are sufficient to describe almost any geometry, however the formulas can be needlessly complicated for certain cases. If we use coordinates which mirror the symmetry of the problem often complicated Cartesian calculations collapse to simple calculations in cylindrical or spherical coordinates. I have spent some effort to derive the gradient, curl and divergence in Spherical and Cylindrical coordinates, please make use of those results where it helps in the problems of this Homework Project.*

PROBLEM 21: Let $f(x,y) = y^2 - x^2$. Plot four level curves of this function in the xy-plane; that is create a contour plot for this function. Calculate the ∇f. Then redraw or copy that plot and add little vectors that illustrate the direction of ∇f though-out representative points in your plot.

PROBLEM 22: Let $f(r,\theta) = r$. Plot four level curves of this function in the xy-plane with polar coordinates r, θ. Calculate ∇f in polar coordinates (see the notes for the formula, or think about Homework Project 18-19 where $s = r,\ \beta = \theta$). Then redraw the contour plot and add little vectors that illustrate the direction of ∇f throughout the plot.

PROBLEM 23: We argued in lecture that ∇F gives the normal vector field to $F(x,y,z) = k$. Use a two-dimensional analogue to that argument to show that ∇f is a field of vectors in the plane that are everywhere orthogonal to the level curves $f(x,y) = k$.

PROBLEM 24: Let

$$f(x,y,z) = \tan^{-1}\left(\frac{y}{x}\right) + \sin^{-1}\left(\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\right) + (x^2+y^2+z^2)^3.$$

Calculate ∇f and $\nabla^2 f$. There is an easy way to do this, and there is a hard way. Choose your path.

PROBLEM 25: Assume that $a, b, c > 0$. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Parametrize this surface via elliptical spherical coordinates β, γ where $0 \leq \beta \leq 2\pi$ and $0 \leq \gamma \leq \pi$ and

$$x = a\cos(\beta)\sin(\gamma), \quad y = b\sin(\beta)\sin(\gamma), \quad z = c\cos(\gamma).$$

Find the vector area element $d\vec{S}$ in terms of these modified spherical coordinates. Find a double integral that gives the surface area of this ellipsoid. Solve the integral in the special case $a = b = c = R$. Assume that $Q_0 \in \mathbb{R}$. Calculate the flux of the vector field $\vec{E} =< 0, 0, Q_0 z >$ through the ellipsoid. If $\vec{E}$ is the the electric field then what is the charge enclosed by the ellipsoid ?

PROBLEM 26: Section 17.3#7, 13, 27.

PROBLEM 27: Section 17.4 #11 and 13.

PROBLEM 28: Section 17.7# 43 and 47.

PROBLEM 29: Section 17.8#6 and 7.

PROBLEM 30: Section 17.9# 10 and 13.