Math 131 Homework Project II: Techniques of Differentiation:

Follow instructions. Be careful to answer all the questions raised in each part. Please turn in neat work with problems clearly labeled and your name on each page. Thanks. Some of these problems are hard. You are not alone. If you get stuck, phone a friend. Or email me, or stop by office hours etc… Start early though, otherwise we may not be able to resolve your questions in time. Also, come to class. I may inadvertently work part of the homework.

PROBLEM 1: [60pts] Calculate the derivative of f for each case. Assume $A, B, C \in \mathbb{R}$ are constants with respect to x. Simplify your answers where appropriate.

a. $f(x) = \sqrt{x}(x+1)$,

b. $f(x) = \frac{\sqrt{x}+7x}{\sqrt{x}} + 3^x$,

c. $f(x) = e^x \sin(x) + \cos(x^2) - 7$

d. $f(x) = (x-2)^A(Ax^2 + Bx + C)\left(x^3 - \frac{1}{\sqrt{x}}\right)^2$

e. $f(u) = Au + B\sin(Cu^2 + \sqrt{u})$

f. $f(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3)$

PROBLEM 2: [15pts] What values must be chosen for m, b such that f is differentiable everywhere given that the function is defined by the rule

$$f(x) = \begin{cases} x^2 & x \leq 2 \\ mx + b & x > 2 \end{cases}$$

PROBLEM 3: [20pts] A differential equation is simply an equation that involves derivatives. For example,

$$y'' + y' - 2y = 0 \qquad \text{(where the notation means } y' = \frac{dy}{dx},\ y'' = \frac{d^2y}{dx^2}\text{)}$$

is a differential equation. A solution to a differential equation is a function that satisfies the differential equation, in other words, if you substitute the function into the differential equation it will work. **Show that $f(x) = e^{\lambda x}$ is a solution provided that the real number λ satisfies a particular quadratic equation. Solve that quadratic equation to find two distinct solutions to the given differential equation.**

(Remark: *in principle differential equations are not much more mysterious then the mathematics we study in calculus I, at some schools a large part of differential equations is taught in the second semester of calculus. This problem shows that we can solve this type of differential equation by solving a corresponding algebraic equation, it turns out this is true for a large class of problems. Just wanted to give you an inkling of what "DEqns" is all about)*

PROBLEM 4: [20pts] Given that g is differentiable calculate the following derivatives,

a. $f(x) = x^2 g(x)$

b. $f(x) = \cos(g^2(x) + g(x))$

PROBLEM 5: [25pts] Suppose that $x = t^2 + 3$ is the equation of motion for a particle travelling in one dimension with position x at time t. Find

a. Velocity $v(t)$,

b. Acceleration $a(t)$,

c. Acceleration when $v = 0$.

PROBLEM 6: [20pts] Find the equation of the tangent line to $y = \tan^{-1}(x)$ at the origin.

PROBLEM 7: [20pts] Using the method of argument presented in my notes (Examples 4.9.4-4.9.8), calculate the derivative of the inverse hyperbolic cosine; that is find a simple formula for $\frac{d}{dx}(\cosh^{-1}(x))$. You will need to use that $\cosh^2(x) - \sinh^2(x) = 1$ at some point in your derivation.

PROBLEM 8: [20pts] Calculate $\frac{dy}{dx}$ given that,

a. $y = x^{e^x + 3}$

b. $x^2 + xy - y^2 = 4$