Math 131 Homework Project IV: Basic Integration and Select Applications:

Follow instructions. Be careful to answer all the questions raised in each part. Please turn in neat work with problems clearly labeled and your name on each page. Thanks. Some of these problems are hard. You are not alone. If you get stuck, phone a friend. Or email me, or stop by office hours etc… Start early though, otherwise we may not be able to resolve your questions in time. Also, come to class. I may inadvertently work part of the homework.

PROBLEM 1: [15pts] Consider the case of one-dimensional motion with acceleration $a = \frac{d^2y}{dt^2} = -g$ where g is a constant. As usual we define velocity $v = \frac{dy}{dt}$ and the position is y. Suppose that the intial position is y_o and the initial velocity is v_o. **Calculate the following items:**

a. **The velocity at time t.**

b. **The position at time t**

c. **The velocity as a function of position.**

For part c. notice that $a = \frac{dv}{dt} = \frac{dy}{dt}\frac{dv}{dy} = v\frac{dv}{dy}$ thus $vdv = -gdy$. (just integrate both sides to get c.)

PROBLEM 2: [25pts] This problem analyzes the total charge flowing out of a discharging capacitor of capacitance C which is connected in series to a resistance R at time zero. If the capacitor has charge Q_o initially it can be shown that the current flow at time $t \geq 0$ is

$$I(t) = \frac{Q_o}{RC}e^{-\frac{t}{RC}}$$

where Q_o, R and C are constants that represent charge, resistance and capacitance respectively. For this problem the so-called "time constant" $\tau = RC$ is said to characterize the time-scale over which the circuit is dynamic. The charge on a discharging capacitor is decreasing so $I = -\frac{dQ}{dt}$ thus $dQ = -Idt$. **Your mission here is to calculate the total charge $Q(t)$ that remains on the capacitor plates for time:**

a. $t = \tau$.

b. $t = 2\tau$

c. $t = 3\tau$

d. $t = 4\tau$

e. $t = 5\tau$

Graph the charge Q as a function of time. Can you see why the time constant gives a good measure of the timescale on which the circuit functions? *(you should use a calculator to find the values of $e^{-1}, e^{-2}, \ldots$ and I suggest using a fraction of Q_o as the vertical scale and τ for the horizontal scale in*

the tQ-graph I request here) [Please ask in lecture if you need further help getting started, but don't wait too long, I'm not helping with this the day before it's due.]

PROBLEM 3: [70pts] Calculate the integrals below, if you make a substitution be sure to explain what substitution was made. If there are bounds be careful to change the bounds as well. I will deduct points if the middle steps are wrong, the answer is the whole calculation not just the last step. The parameters a, b, c are to be treated as constants with respect to the variable of integration.

a. $\displaystyle\int_{-1}^{1} |x|\ dx$

b. $\displaystyle\int \sin(\theta)e^{3\cos(\theta)}\ d\theta$

c. $\displaystyle\int \left(\frac{2x+1}{x^2+1}\right) dx$

d. $\displaystyle\int_{0}^{1} 2y(ay^2+1)^5\ dy$

e. $\displaystyle\int \left(\sin(ax)+\cos(bx)+xe^{cx^2}\right)dx$

f. $\displaystyle\int_{1}^{2} \frac{dt}{a-bt}$

g. $\displaystyle\int \frac{e^{2x}dx}{\sqrt{3-e^{4x}}}$

PROBLEM 4: [45pts] Calculate the area bounded by the following curves. Your solution should include a sketch of the area, typical infinitesimal rectangle(s) and any algebra needed to justify where the intersection points occur. (an integral with no graph is not a valid solution)

a. $y = x + 1$ and $y = 4 - x^2$
b. $y = x^2$ and $y = |x|$ and $x = -1$ and $x = 2$
c. $x = y^2$ and $x = 2 - y^2$

PROBLEM 5: [45pts] Calculate the volumes described below. Your solution should include a sketch of the volume, typical infinitesimal cross-section(s) or cylindrical shell, and any algebra needed to justify where the intersection points occur. (an integral with no graph is not a valid solution)

a. right circular cone of radius R and height h
b. volume obtained from rotating the area bounded by $y = x$ and $y = x^2$ about $x = 3$
c. volume obtained from rotating the area bounded by $y = x$ and $y = x^2$ about $y = 2$
d. volume obtained from rotating the area bounded by $y = \sin(x)$ and $x = 2\pi$, $x = 5\pi/2$ about the y-axis. (you will need to know that $\int x\sin(x)dx = -x\cos(x) + \sin(x) + c$)

(If you remind me I will draw pictures to help with Problems 4 and 5 in lecture)