Math 131 Homework Project I: Limits and Functions:

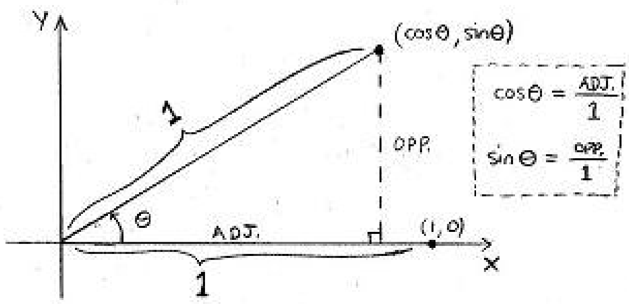
Follow instructions. Be careful to answer all the questions raised in each part. Please turn in neat work with problems clearly labeled and your name on each page. Thanks. Some of these problems are hard. You are not alone. If you get stuck, phone a friend. Or email me, or stop by office hours etc… Start early though, otherwise we may not be able to resolve your questions in time. Also, come to class. I may inadvertently work part of the homework.

PROBLEM 1: [30pts] Find the lowest degree polynomial function f whose graph goes through the points $(1,6),\ (-1,0),\ (0,0),\ (2,0)$. Then give an example of a higher degree polynomial which also goes through those points and $(3,0)$. *Hint: if it takes two points to determine a line then what do four points determine?*

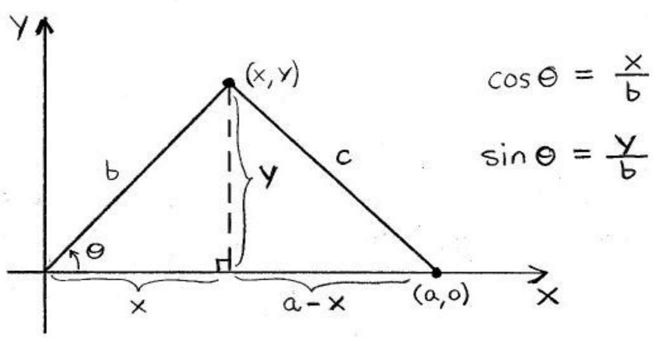
PROBLEM 2: [30pts] Let $k \in \mathbb{R}$. Solve $k = -e^x$ for x. State what condition on k must be met in order that this solution exist as a real number.

PROBLEM 3: [180pts] Trigonometry refresher. Assume that $a, b, c, \theta, \alpha, \beta, \gamma \in \mathbb{R}$.

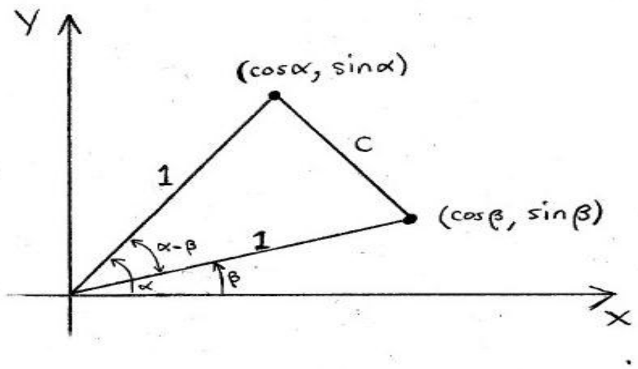
a. Prove that $\cos^2(\theta) + \sin^2(\theta) = 1$ using the picture below plus the Pythagorean Theorem.



b. The Law of Cosines tells us how the sides of a triangle are related to a particular angle. This generalizes the Pythagorean Theorem for triangles which are not right triangles (meaning they do not have a 90 degree angle). Use the picture to prove that $c^2 = a^2 + b^2 - 2ab\cos(\theta)$.



c. In section 4.11 of my notes I provide a sneaky quasi-rigorous argument for proving the "adding angles" formula for cosine. That formula implies $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$. Your mission here is to provide an alternate argument based on the picture below. You will need to use the law of cosines from part b.



d. Given that $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$ find the corresponding formula for $\cos(\alpha + \beta)$ with the help of the fact that cosine is an even-function while sine is an odd- function.

e. We know that $\cos^2(\gamma) + \sin^2(\gamma) = 1$ from part a. Let $\gamma = \alpha + \beta$ and use the adding angles formula for cosine to show that $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$.

f. Calculate the formula for $\sin(\alpha - \beta)$ from the result of e. Again use that sine is an odd-function and cosine is an even-function to find the corresponding formula.

PROBLEM 4: [60pts] Calculate the following limits. Show work, justify each step. Feel free to use Theorem 3.3.1 in my notes.

a. $\lim_{x \to -2}(3x^4 + 2x^2 - x + 1)$

b. $\lim_{x \to 16}\left(\dfrac{4 - \sqrt{x}}{16x - x^2}\right)$

c. $\lim_{t \to 0}\left(\dfrac{1}{t} - \dfrac{1}{t^2 + t}\right)$

PROBLEM 5: [50pts] Carefully prove that $\lim_{x \to 2}(2x) = 4$ using the $\epsilon - \delta$ definition of the limit.

PROBLEM 6: [50pts] Suppose that $x = t^2 + 3$ is the equation of motion for a particle travelling in one dimension with position x at time t. Find the equation for the tangent line through $(1, 4)$ in the tx-plane *(in this context the independent variable is t and the dependent variable is x so the tx-plane has time going horizontally)*. Explain what the slope of the tangent line means physically. Contrast that to the average velocity from time $t = 0$ to $t = 1$. Graph the function and illustrate both the secant line which corresponds to the average velocity and the tangent line through $(1, 4)$.