Working together is encouraged, share ideas not calculations. Explain your steps. I will collect some subset of these problems. A page to write answers on will be distributed in class the day before the Mission is due.

Problem 361 Please read Chapter 5 of the Lecture Notes.

Problem 362 Current $I = \frac{dQ}{dt}$ where Q is the charge. Calculate the net change in charge over $0 \leq t \leq 3$ given that $I(t) = 10(1 - e^{-2t})$.

Problem 363 If a pet-educator throws poodles out a window at the rate $\frac{dP}{dt} = 2t + 1$ from $t = 1$ to $t = 3$ then find the net number of poodles thrown out the window from time $t = 1$ to time $t = 3$. In case you have not seen it, you might watch: this clip from Weird Al's classic movie UHF.

Problem 364 We define $f_{avg} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$ for the average of $f(x)$ on $a \leq x \leq b$. Find f_{avg} for $f(x) = x^2$ on $0 \leq x \leq 2$.

Problem 365 Suppose f is continuous on $[a, b]$ and $f(x) \geq 0$ for simplicity of discussion. Solve $\int_a^b f(x)\,dx = \int_a^b c\,dx$ for c. What can you say about f_{avg} in view of this calculation.

Problem 366 Find the signed-area bounded between $y = \sin(2x)$ and the x-axis where $0 \leq x \leq \pi$.

Problem 367 Find the area bounded between $y = \sin(2x)$ and the x-axis where $0 \leq x \leq \pi$. Your solution should include a graph which motivates your calculation.

Problem 368 Find the area bounded by $y = x^2 - 9$, the x-axis, $x = -4$ and $x = 4$. Your solution should include a graph which motivates your calculation.

Problem 369 Find the area bounded by $y = 2x - 10$ and $x = 10 - y^2$. Your solution should include a graph which motivates your calculation.

Problem 370 Find the area bounded by $y = \sqrt{x - 1}$ and $y = x - 1$. Your solution should include a graph which motivates your calculation.

Problem 371 Find the velocity and position x at time t given the acceleration $a(t) = t + 4$. Write your answer in terms of the inital velocity v_o and position x_o.

Problem 372 Suppose the initial position is $x = 0$ and the initial velocity is $v = 2$ at time $t = 1$. If the acceleration is given by $a(t) = t^3 - 1$ for $t \geq 1$ then find the velocity and position at time $t \geq 1$.

Problem 373 Suppose the velocity at time t is given by $v(t) = 10 + \sin(2t)$. If the initial position at time $t = 0$ is $x = 2$ then find the position and acceleration at time $t \geq 0$.

Problem 374 Suppose $v(t) = t^2 - 4$. Find the distance travelled during the time interval $[0, 4]$.

Problem 375 If $3\int_0^a e^x dx = \int_0^b e^x dx$ then how are a and b related ? Solve for b as a function of a.

Problem 376 Find area bounded by $y = x + 1$ and $y = 9 - x^2$ and $x = -1$ and $x = 2$. Include a sketch of the area as well as your typical infinitesimal approximating rectangle.

Problem 377 Find area bounded by $y = x$ and $y = x^2$. Include a sketch of the area as well as your typical infinitesimal approximating rectangle.

Problem 378 Find area bounded by $y = \sqrt{x + 3}$ and $y = \frac{1}{2}(x + 3)$. Include a sketch of the area as well as your typical infinitesimal approximating rectangle.

Problem 379 Find area bounded by $y = x^2$ and $y^2 = x$. Include a sketch of the area as well as your typical infinitesimal approximating rectangle.

Problem 380 Find area bounded by $y = 12 - x^2$ and $y = x^2 - 6$. Include a sketch of the area as well as your typical infinitesimal approximating rectangle.

Problem 381 Find area bounded by $x = 2y^2$ and $x = 4 + y^2$. Include a sketch of the area as well as your typical infinitesimal approximating rectangle.

Problem 382 Find area bounded by $x = 1 - y^2$ and $x = y^2 - 1$. Include a sketch of the area as well as your typical infinitesimal approximating rectangle.

Problem 383 Find area of triangle with vertices $(0,0), (2,1), (-1,6)$. Include a sketch of the area as well as your typical infinitesimal approximating rectangle.

Problem 384 Find a value b such that $y = b$ divides the area bounded by $y = x^2$ and $y = 4$ into two equal parts.

Problem 385 Find the volume of solid formed by revolving the area bounded by $y = -x/2 + 2$ and $y = 0$ and $x = 1$ and $x = 2$ around the x-axis. Also, find the volume if we instead rotate around the $y = -1$ axis.

Problem 386 Find the volume of solid formed by revolving the area bounded by $y = 1/x$ and $x = 1$ and $x = 2$ and $y = 0$ around the x-axis.

Problem 387 Find the volume of solid formed by revolving the area bounded by $x = 2\sqrt{y}$ and $x = 0$ and $y = 9$ around the y-axis. Also, find the volume if we rotate around the $x = 8$ axis.

Problem 388 Find the volume of solid formed by revolving the area bounded by $y = x^3$ and $y = x$ and $x \geq 0$ around the x-axis.

Problem 389 Find the volume of solid formed by revolving the area bounded by $y^2 = x$ and $x = 2y$ around the y-axis.

Problem 390 Find the volume of the cap of a sphere of radius R where the cap is distance h from the center of the sphere.

Problem 391 Find the volume of a rectangular pyramid with base with width b and length $2b$ and a height h.

Problem 392 Use the method of cylindrical shells to find the volume of the solid generated by rotation of the area bounded by $y = x^2$ for $0 \leq x \leq 2$ and $y = 4$ and $x = 0$ around the y-axis. Include a diagram to explain your calculation.

Problem 393 Use the method of cylindrical shells to find the volume of the solid generated by rotation of the area bounded by $y = 4(x-2)^2$ and $y = x^2 - 4x + 7$ around the y-axis. Include a diagram to explain your calculation.

Problem 394 Use the method of cylindrical shells to find the volume of the solid generated by rotation of the area bounded by $x = 1 + y^2$ and $x = 0$ and $y = 1$ and $y = 2$ around the x-axis. Include a diagram to explain your calculation.

Problem 395 Use the method of cylindrical shells to find the volume of the solid generated by rotation of the area bounded by $y = x^3$ and $y = 8$ and $x = 0$ around the x-axis. Include a diagram to explain your calculation.

Problem 396 In physics the force F in a one-dimensional problem with coordinate x is called **conservative** if there exists a potential energy function U for which $F = -\frac{dU}{dx}$. If the coordinate was y then this becomes $F = -\frac{dU}{dy}$ etc. Find a[1] potential energy function given:

(a.) $F = -kx$ where k is a constant and the coordinate is x,

(b.) $F = -mg$ where m, g are constants and the coordinate is y,

(c.) $F = -\dfrac{Gm_1m_2}{r^2}$ where G, m_1, m_2 are constants and the coordinate is r.

Problem 397 In physics, the net-force F in a one-dimensional problem with coordinate x must satisfy Newton's Second Law $F = ma$ where $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$ and we assume the mass m is constant. Let $T(v) = \frac{1}{2}mv^2$ define the **kinetic energy**. Prove $\displaystyle\int_{x_1}^{x_2} F(x)dx = T(v_2) - T(v_1)$. This result is known as the **work-energy theorem**.

Problem 398 Let F be a one-dimensional conservative force with $F = -\frac{dU}{dx}$ and let $T(v) = \frac{1}{2}mv^2$ as in the previous problem. If F is the net-force and $E(x,v) = U(x) + T(v)$ then prove that energy E is conserved along the equations of motion. That is, show that $\frac{dE}{dt} = 0$ for solutions of Newton's Second Law.

[1] notice there is a freedom to set $U = 0$ wherever we so desire, this is an example of gauge freedom

Problem 399 Work done by F in the x-direction over the interval $[x_1, x_2]$ is defined by $W = \int_{x_1}^{x_2} F(x)\,dx$. Let F_o be a constant. Calculate the work done by $F = F_o$ from $x = x_1$ to $x = x_2$.

Problem 400 Work done by F in the x-direction over the interval $[x_1, x_2]$ is defined by $W = \int_{x_1}^{x_2} F(x)\,dx$. Calculate the work done by $F = -kx$ from $x = x_1$ to $x = x_2$.